Math 241 — Calculus III

Midterm 2 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Mar. 3, 3-5pm Anjali, Jesse, Nidhi
Session 2: Mar. 6, 3-5pm Aditya, David, Matthew

Can’t make it to a session? Here’s our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host, as well as posted in the queue during the session.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Good luck with your exam!
1. Calculate the following derivatives:

(a) Find \( \frac{df}{dt} \) for \( f(x, y) = xe^{xy}, \) \( x(t) = t^2, y(t) = \frac{1}{t} \)

(b) Find \( f_t \) for \( f(x, y) = 2xy, \) \( x(s, t) = st, y(s, t) = s^2t^2 \)

2. Show that \( f(x, y) = y^2e^{xy} \) is differentiable at \((0, 2)\) and use linear approximation to find the value of \( f(0.1, 2.1) \).
3. A tiny spaceship is orbiting a path given by $x^2 + y^2 = 4$. The solar radiation at a point $(x, y)$ in the plane is $f(x, y) = xy + 2y$.

Use the method of Lagrange multipliers to find the maximum value and minimum value of solar radiation experienced by the tiny spaceship in its orbit.

4. For each equation below, match it with the corresponding graph.

(A) $z = \ln(x^2 + y^2)$
(B) $z = \sin(xy)$
(C) $z = e^x \cos(y)$
(D) $z = \frac{1}{1+x^2y^2}$
5. Let $f(x,y)$ be a differentiable function on the disk $D = x^2 + y^2 \leq 400$, where:
(a) $f(x, y) = 19$ for every point on the boundary of the disk $x^2 + y^2 = 400$
(b) $f(0, 0) = 7$
(c) $f(x, y)$ has only one critical point which is at (-1,2)

Decide which statement is true:

A) $f(-1, 2) > 7$
B) $f(-1, 2) < 7$
C) $f(-1, 2) = 7$
D) Not enough information is given

6. Consider the function $f(x, y) = x^3 + y^3 + 3xy$

(a) The critical points of $f$ are $(0, 0)$ and $(-1, -1)$. Classify them into local minima, local maxima and/or saddle points

(b) Based on your answer in (a), identify the correct contour diagram of $f$
7. What is the partial derivative of \( f(x, y, z) = e^x \sin(yz)z^3 \ln(y) \) with respect to \( x \).

8. Consider a function \( f(t) = f(x(t), y(t), z(t)) = xyz - z^2 \), where \( x(t), y(t), z(t) \) are defined as followed:

\[
\begin{align*}
  x(t) &= 2t^2 + 1 \\
  y(t) &= 3 - \frac{1}{t} \\
  z(t) &= 3
\end{align*}
\]

Find the following values:

(a) \( f_z(3,1,2) \)

(b) \( \frac{d}{dt} x(t = 0) \)

(c) \( \frac{d}{dt} f(t = 1) \)
9. Consider the limit

$$\lim_{(x,y) \to (0,0)} \frac{y^4 \cos^2 x}{x^4 + y^4}$$

Does this limit exist? If so, what is its value? Justify your answer.

10. If $f(x, y, z) = xye^z$, find the gradient of $f$ and the directional derivative at $(2,5,0)$ in the direction of $\vec{v} = 2\hat{i} - \hat{j} + \hat{k}$.