The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Nov 2, 6-8pm: Danny, Pieter, Matthew
Session 2: Nov 3, 7-9pm: David, Nidhi, Sean

Can’t make it to a session? Here’s our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!
1. The vector field \( \vec{F} = (2xy + 2x + y^2, 2xy + 2y + x^2) \) is conservative. Find a potential function \( f \) for \( \vec{F} \) (a function with \( \nabla f = \vec{F} \)).

2. A particle moves along the upper part of an ellipse in the \( xy \)-plane that has its center at the origin with semi-major and semi-minor axes \( a = 4 \) and \( b = 3 \), respectively. Starting at \((a, 0)\) and ending at \((-a, 0)\) and subject to the following force field, what is the total work done?

\[
\vec{F} = (3x - 4y + 2z)\hat{i} + (4x + 2y - 3z^2)\hat{j} + (2xz - 4y^2 + z^3)\hat{k}
\]

3. Compute the work done by the force field \( \vec{F} = x\hat{i} + 3xy\hat{j} - (x + z)\hat{k} \) as a particle moves in a line segment from \((1, 4, 2)\) to \((0, 5, 1)\).
4. Let \( r(t) = (\sin(t), \cos^2(t)), 0 \leq t \leq 2\pi \). Which graph below represents this curve?

\[ \begin{align*}
A & \quad \text{y-axis orientation} \\
B & \quad \text{unit circle} \\
C & \quad \text{hyperbola} \\
D & \quad \text{linear function}
\end{align*} \]

5. Consider the following vector fields \( \vec{F}(x, y, z) \). Are they conservative? If so, find a function \( f(x, y, z) \) so that \( \nabla f = \vec{F} \). If not, justify your response.

(a) \( \vec{F}(x, y, z) = (yz, xz, xy + 2z) \)

(b) \( \vec{F}(x, y, z) = (y + e^x, x - \cos y, 4 + z) \)

(c) \( \vec{F}(x, y, z) = (y, z^2, x) \)
6. Find the work done by the force field below in moving an object from (1,1) to (2,4) (HINT: Check if the vector field is conservative).

\[ \vec{F}(x, y) = \langle 6y^2, 9x\sqrt{y} \rangle \]

7. Compute the double integral over the indicated rectangle. Confirm your answer by switching the order of integration and recomputing.

\[
\iint_R (2x - 4y^3) \, dA \quad R = [-5, 4] \times [0, 3]
\]

8. Making an appropriate change of variables, compute the following double integral over the region bound by a circle of radius 2 and a circle of radius 5.

\[
\iint_D e^{x^2+y^2} \, dA
\]