The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive.

In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Oct 27, 5-7pm Justin and Jose

Can’t make it to a session? Here’s our schedule by course:

https://care.grainger.illinois.edu/tutoring/schedule-by-subject

Solutions will be available on our website after the last review session that we host.

Step-by-step login for exam review session:

1. Log into Queue @ Illinois: https://queue.illinois.edu/q/queue/792
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”

Please be sure to follow the above steps to add yourself to the Queue.

Good luck with your exam!
1. Find the absolute minimum y-value of the given function:

\[ y = \frac{2x}{\sqrt{x - 81}} \]

Domain of the function: \( x > 81 \).

\[ y' = \frac{2\sqrt{x - 81} - 2x(\frac{1}{2})(x - 81)^{-\frac{1}{2}}}{(\sqrt{x - 81})^2} \]

\[ y' = \frac{2\sqrt{x - 81} - \frac{x}{\sqrt{x - 81}}}{x - 81} \]

\[ y' = \frac{2(x - 81) - x}{(x - 81)(\sqrt{x - 81})} \]

\[ y' = \frac{x - 162}{(x - 81)\sqrt{x - 81}} \]

There is an absolute max at \( x = 162 \) and that amount is 36, which can be found by plugging 162 into the function for y.

2. The function \( f(x) = 10x^3 - 20x + 1 \) has one root in the interval \([1, 2]\). In order to approximate this root, begin with an initial estimate of \( x_1 = 2 \) and use Newton’s Method to obtain a second estimate \( x_2 \), then write it in decimal form.

\[ f'(x) = 30x^2 - 20 \]

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \]

\[ x_2 = 2 - \frac{f(2)}{f'(2)} \]

\[ x_2 = 2 - \frac{41}{100} = 2 - 0.41 = 1.59 \]
3. A farmer wishes to fence off three identical adjoining rectangular pens as in the diagram shown, but only has 600 feet of fencing available. Determine the values for x and y which will maximize the total area enclosed by these three pens.

\[
\text{Area} = 3xy \\
\text{Perimeter} = 600 = 6x + 4y
\]

\[
4y = 600 - 6x \\
y = 150 - 1.5x
\]

Area is now equivalent to:

\[
\text{Area} = 3x \times (150 - 1.5x) \\
\text{Area} = 450x - 4.5x^2
\]

Now, we must maximize A for x in the range of (0, 100)

\[
0 = 450 - 9x \\
9x = 450 \\
x = 50
\]

Check for the values of A’:

\[
\begin{array}{cccc}
+ & + & + & - & - & - \\
0 & 50 & 100
\end{array}
\]

So we can see that there is an absolute maximum at \( x = 50 \). Evaluate \( y \) at \( x = 50 \)

\[
y = 150 - 1.5 \times (50) \\
y = 75
\]

\[
\text{Area} = 3 \times 50 \times 75 = 11,250 \text{ ft}^2
\]
4. A function \( f(x) \) has the first derivative \( f'(x) = e^{0.5x}(10x - 60) \)

(a) Upon which interval is \( f(x) \) increasing?

\[
\begin{array}{c|c|c|c|c|c|c}
& & & & + & + & + \\
& & & 0 & 6 & & \\
& & & & & & \\
\end{array}
\]

Based off of this, we can say \( f \) is increasing on the interval \((6, \infty)\). So the answer is \([6, \infty)\).

(b) Upon which interval is the graph of \( f(x) \) concave down?

\[
f'(x) = 0.5e^{0.5x}(10x - 60) + e^{0.5x} \cdot 10
\]

\[
f'(x) = e^{0.5x}(0.5(10x - 60) + 10)
\]

\[
f'(x) = e^{0.5x}(5x - 20)
\]

Values of \( f'(x) \):

\[
\begin{array}{c|c|c|c|c|c|c}
& & & + & + & + \\
& & & 0 & 4 & & \\
& & & & & & \\
\end{array}
\]

The function is concave down on the interval \((-\infty, 4)\)

5. Use a linear approximation to estimate the value of

\[
\ln\left(\frac{95}{100}\right)
\]

Write your answer as either a simplified fraction or a decimal value.

\( f(x) = \ln(x) \), and we need to find the tangent line at nearby point \( x = 1 \) to approximate this.

\[
f'(x) = \frac{1}{x} \implies f'(1) = 1, \text{ so the approximation is } L(x) = 1x + b. \text{ We know that the original function evaluates to zero at } x = 1, \text{ so this must do the same. We can use that to solve for } b.
\]

\[
L(1) = 0 \implies b = -1, \text{ so } L(x) = x - 1
\]

Near \( x = 1, f(x) \) is approximately equal to \( L(x) \) so, \( \ln\left(\frac{95}{100}\right) \approx L\left(\frac{95}{100}\right) = \frac{95}{100} - 1 = \frac{-1}{20} = -0.05 \)
6. Estimate the $x$-value for the point of intersection on the graphs of $y = x^3 + 2x$ and $y = 2x + 4$ using Newton’s Method with an initial estimate of $x_1 = 1$. You should use this method twice to obtain estimates $x_2$ and $x_3$.

$$x^3 + 2x = 2x + 4$$
$$x^3 - 4 = 0$$

Apply Newton’s Method to estimate a root of $f(x) = x^3 - 4$ (note $f'(x) = 3x^2$). Use the given $x_1$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{(1)^3 - 4}{3(1)^2} = 1 - \frac{-3}{3} = \frac{2}{3}$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{(2)^3 - 4}{3(2)^2} = 2 - \frac{4}{12} = \frac{5}{3}$$

7. The acceleration due to gravity near the surface of some planet is $-8 \text{ m/s}^2$. An object is thrown upward from the surface of this planet, and 12 seconds later it has fallen back to the surface. It is known that its height above the surface as a function of time is described by this equation:

$$s(t) = -4t^2 + 48t$$

(a) What is the velocity of this object 2 seconds after being thrown?

Begin by differentiating the position equation with respect to time.

$$s'(t) = -8t + 48$$

$$s'(2) = -8(2) + 48 = 32 \text{ m/s}$$

(b) What is the maximum height of this trajectory?

At the trajectory’s maximum height, the velocity will be zero.

$$s'(t_{s_{\text{max}}}) = 0 = -8t_{s_{\text{max}}} + 48 \implies t_{s_{\text{max}}} = 6$$

$$s(6) = -4(6)^2 + 48(6) = 144 \text{ m}$$