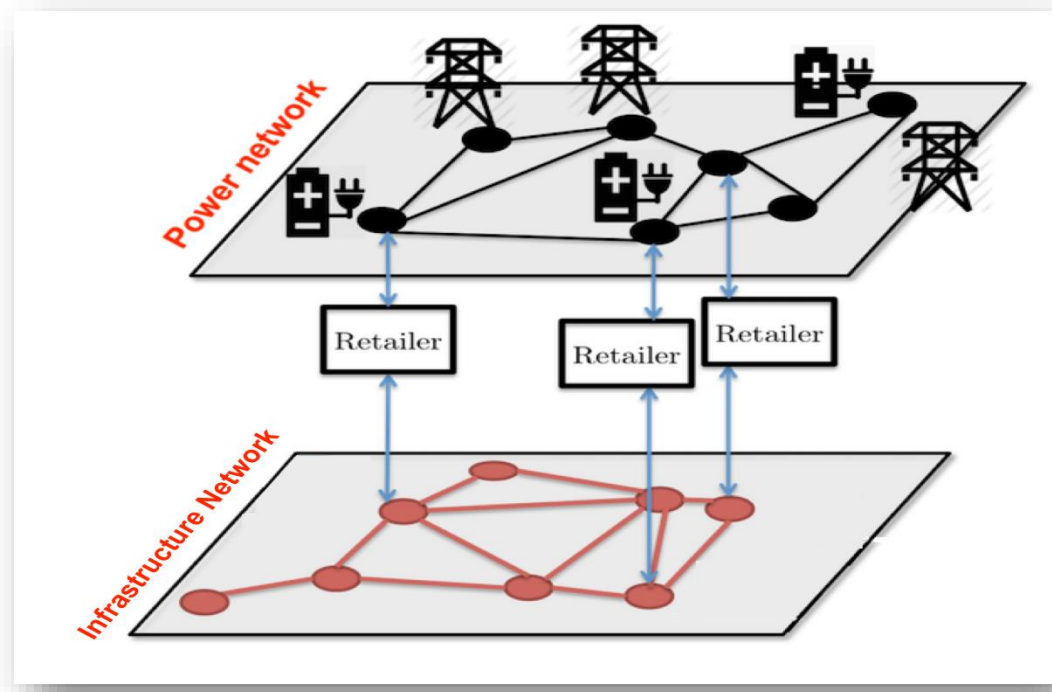


GOALS

Many large infrastructure networks are **COUPLED** with power networks!



- Examples of coupled infrastructure networks
 - Gas network
 - Water Network
 - Electric vehicles (EVs)
 - Data centers

• **Goal:** to investigate the way coupled infrastructure can address the issue of resilience in concert

- We start by studying congestion and possible wide area oscillations
 - In an ideal world: Maximum Social Surplus solution (or Social Optimum (SO)) finds the optimum settings across the infrastructures (**most secure**)
 - Can the systems operate and recover separately and still be secure?

BACKGROUND STUDIES / FORMULATION

MAX. SOCIAL SURPLUS SOLUTION w/ COUPLED OPTIMIZATION

- Utopia-like, when the infrastructure and IPSO are fully cooperating.

min $J_{\text{Power}}(\mathbf{g}) + J_{\text{Infrast./Gas}}(\boldsymbol{\lambda})$ IPSO solves economic dispatch, with the power demand modulated by services on infrastructure network.

s.t. Power system constraints:

$$\begin{aligned} \mathbf{1}^T(\mathbf{e} + \mathbf{u} - \mathbf{g}) &= 0, && \rightarrow \text{[balance]} \\ \mathbf{H}(\mathbf{e} + \mathbf{u} - \mathbf{g}) &\preceq \mathbf{c}, && \rightarrow \text{[line flow limit]} \\ \mathbf{0} &= \mathbf{f}(\mathbf{e}, \boldsymbol{\lambda}) && \rightarrow \text{[infrast. to elect. demand map]} \end{aligned}$$

Operation constraints for infrastructure

EXAMPLES OF INFRASTRUCTURE SYSTEMS

Electric Vehicle Network

Possible paths: \mathcal{Q}

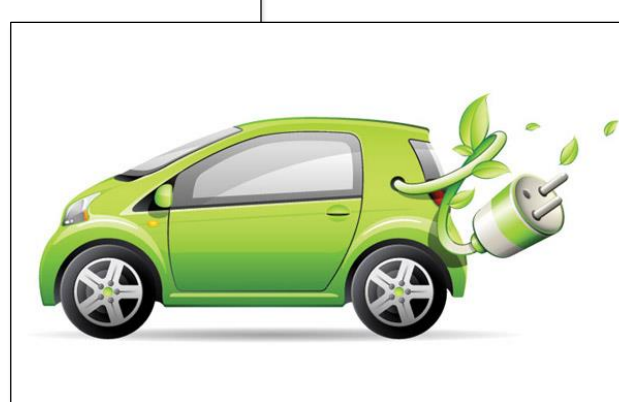
- Decision: **flows** of EVs $\boldsymbol{\lambda} \in \mathbb{R}^{|\mathcal{E}|}$

Utility function: $J_{\text{EV}}(\boldsymbol{\lambda}) = \boldsymbol{\lambda}^T \mathbf{s}(\boldsymbol{\lambda})$

Coupling constraints: $\mathbf{f}(\mathbf{e}, \boldsymbol{\lambda}) = \mathbf{E}\boldsymbol{\lambda} - \mathbf{e}$

Operation constraints:

$$\begin{aligned} \text{[non-negativity]} \quad \mathbf{d}_q &\geq \mathbf{0} && \text{[individual demand]} \quad \mathbf{1}^T \mathbf{d}_q = a_q && \text{[map from path flow to total flow]} \quad \boldsymbol{\lambda} = \sum_{q \in \mathcal{Q}} \mathbf{A}_q \mathbf{d}_q \end{aligned}$$



Gas Pipeline Network

- Decision variables: $\boldsymbol{\lambda} = (\mathbf{d}, \boldsymbol{\alpha}, \boldsymbol{\rho}, \boldsymbol{\phi})$

Gas withdrawal \mathbf{d} (controllable)
Compressors $\boldsymbol{\alpha}$
Pressure $\boldsymbol{\rho}$
Mass flows $\boldsymbol{\phi}$



Utility function: $J_{\text{Gas}}(\boldsymbol{\lambda}) = \sum_{(i,j) \in \mathcal{E}} |\phi_{\pi_e(ij)}| (\max\{\alpha_{ij}, 1\}^{2m} - 1)$

Coupling constraints: $\mathbf{f}(\mathbf{e}, \boldsymbol{\lambda}) = q_0 + q_1 \mathbf{e} + q_2 \mathbf{e}^2 - \mathbf{d}$

Operation constraints:

$$\begin{aligned} \text{[gas withdrawal demand]} \quad \mathbf{d} &= \mathbf{A}_d \boldsymbol{\phi} && \text{[compressors limit]} \quad \mathbf{1} \leq \boldsymbol{\alpha} \leq \boldsymbol{\alpha}^{max} && \text{[pressure limit of pipelines]} \quad \boldsymbol{\rho}^{min} \leq \boldsymbol{\alpha} \boldsymbol{\rho} \leq \boldsymbol{\rho}^{max} \end{aligned}$$

$$\text{[steady state of gas flow equations]} \quad \mathbf{B}_s(\boldsymbol{\alpha}) \mathbf{s} + \mathbf{B}_d(\boldsymbol{\alpha}) \boldsymbol{\rho} = -\boldsymbol{\Lambda} \mathbf{K} \mathbf{g}(\boldsymbol{\phi}, |\mathbf{B}_s(\boldsymbol{\alpha})| \mathbf{s} + |\mathbf{B}_d(\boldsymbol{\alpha})| \boldsymbol{\rho})$$

MULTIPLE INFRASTRUCTURE OPERATORS

- Assume **multiple** retailers operating the infrastructure.
- The retailers individually choose their flow on the **same** network.

$$\boldsymbol{\lambda}^r = \arg \min_{\boldsymbol{\lambda}^r \in \mathcal{F}^r} J(\hat{\boldsymbol{\lambda}}^r; \boldsymbol{\lambda}^{-r}; \mathbf{p})$$

Retailer r has only control of its **virtual flow** and has its **private demand** to satisfy.

SECURITY CONCERNS:

- Static Model Oscillations? \rightarrow Any Nash Equilibrium (NE)?
- Dynamic Model? \rightarrow Resonant frequencies for the pipelines!

RESEARCH RESULTS – STEADY STATE MODEL

EXISTENCE OF NASH EQUILIBRIUM & UNIQUENESS

- For the special case of EV network, the multi-retailer problem is a **special case** of the competition game in network routing.
- Therefore an NE will always exist^a. Furthermore, the NE is unique^b if

$$s_a(\lambda_a) \propto (\lambda_a)^\alpha \quad \text{with } \alpha \leq (3R - 1)/(R - 1)$$

As number of retailers, R , increases, the NE may be non-unique for large α .

^a A. Orda, R. Rom, and N. Shimkin, "Competitive routing in multiuser communication networks," IEEE ToN, 1993.
^b E. Altman, T. Basar, T. Jimenez, and N. Shimkin, "Competitive routing in networks with polynomial costs," IEEE TAC, 2002.

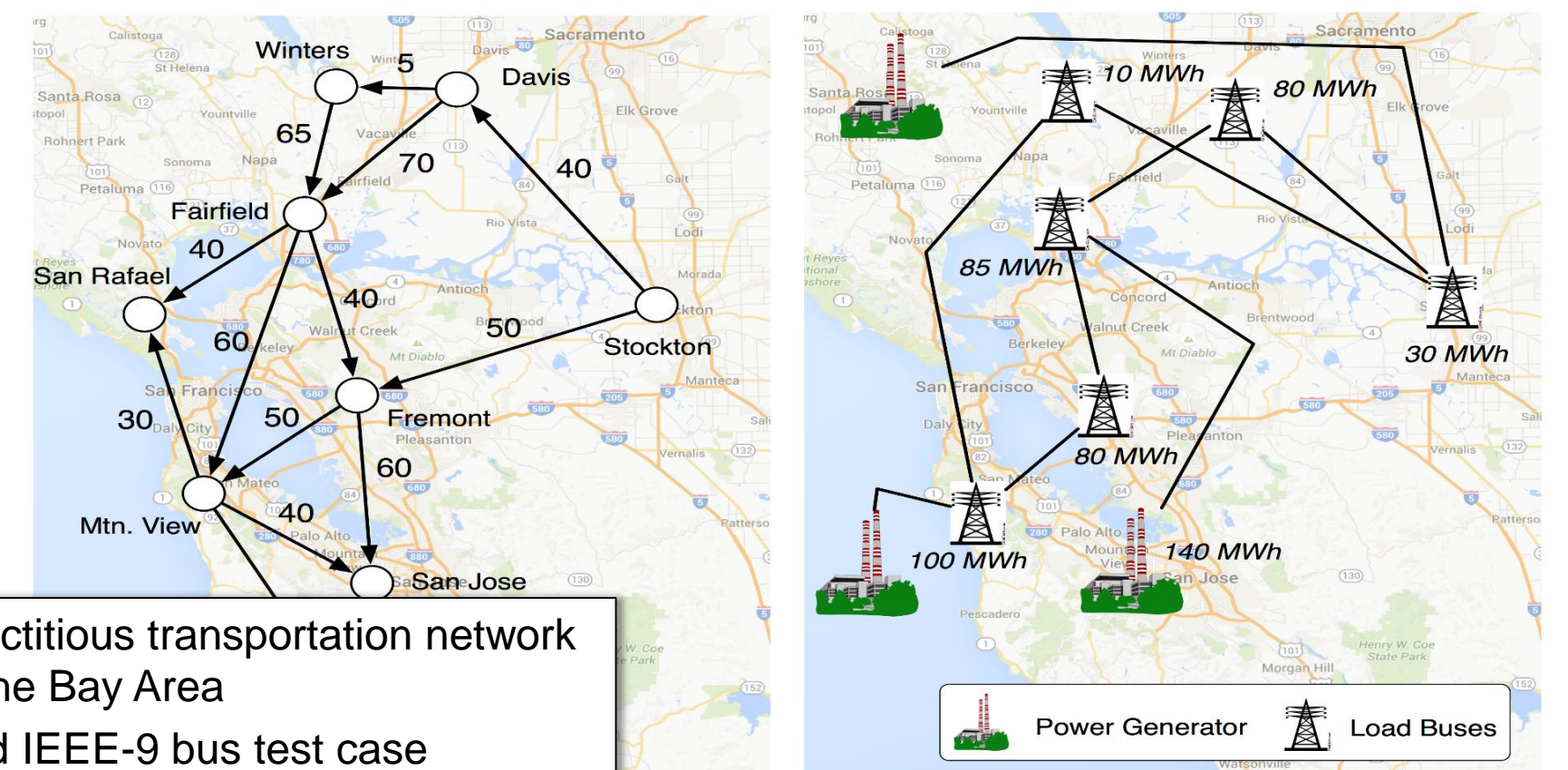
FINDING THE NE VIA BI-LEVEL OPTIMIZATION

- **Aim:** find optimal pricing for IPSO that leads to a Nash Equilibrium.

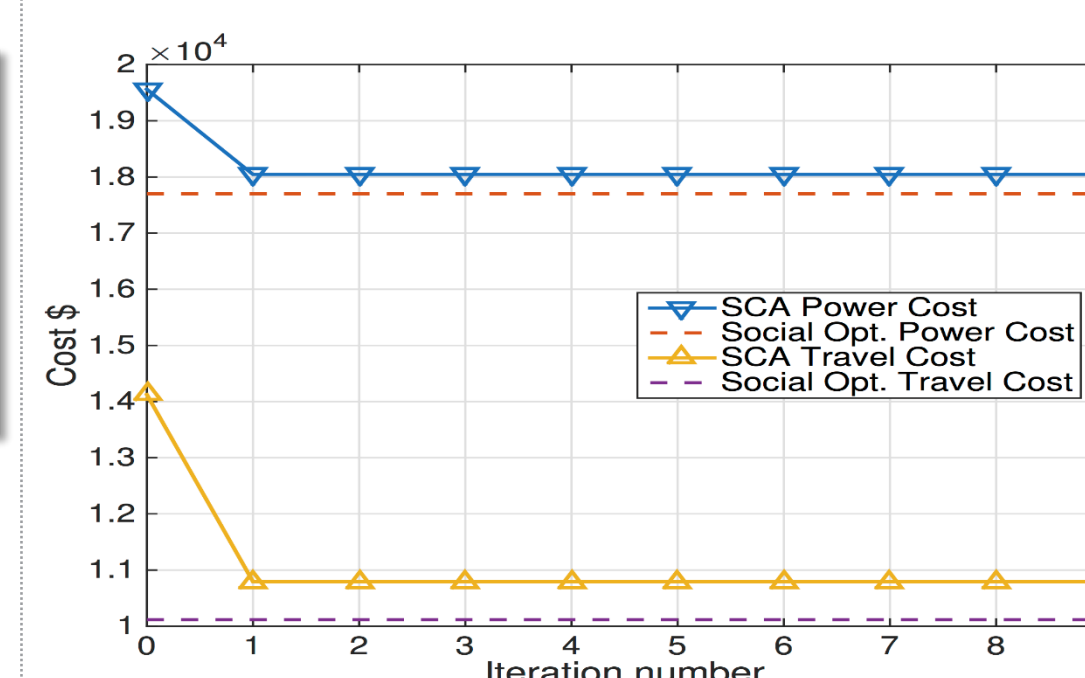
$$\begin{aligned} \min_{\mathbf{g}, \mathbf{p}} \quad & \mathbf{1}^T \mathbf{c}(\mathbf{g}) \\ \text{s.t.} \quad & \mathbf{g} \geq \mathbf{0}, \quad \mathbf{p} = \mathbf{H}^T \boldsymbol{\mu} + \boldsymbol{\gamma} \mathbf{1}, \\ & \boldsymbol{\gamma} : \mathbf{1}^T (\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) = \mathbf{0}, \quad \boldsymbol{\mu} : \mathbf{H}(\mathbf{d} + \boldsymbol{\ell} - \mathbf{g}) \leq \mathbf{m}, \\ & \mathbf{d} = \mathbf{M} \sum_{r \in \mathcal{R}} \boldsymbol{\lambda}^r, \\ & \forall r \in \mathcal{R} : \boldsymbol{\lambda}^r = \arg \min_{\boldsymbol{\lambda}^r \in \mathcal{F}^r} J(\tilde{\boldsymbol{\lambda}}^r; \boldsymbol{\lambda}^{-r}; \mathbf{p}). \end{aligned}$$

Bi-level optimization problem

- Leader-Follower structure --- leader: IPSO, follower: retailers
- The bi-level problem can be solved as a mixed integer program.
- Invokes convex approximation to handle the non-convex constraints.



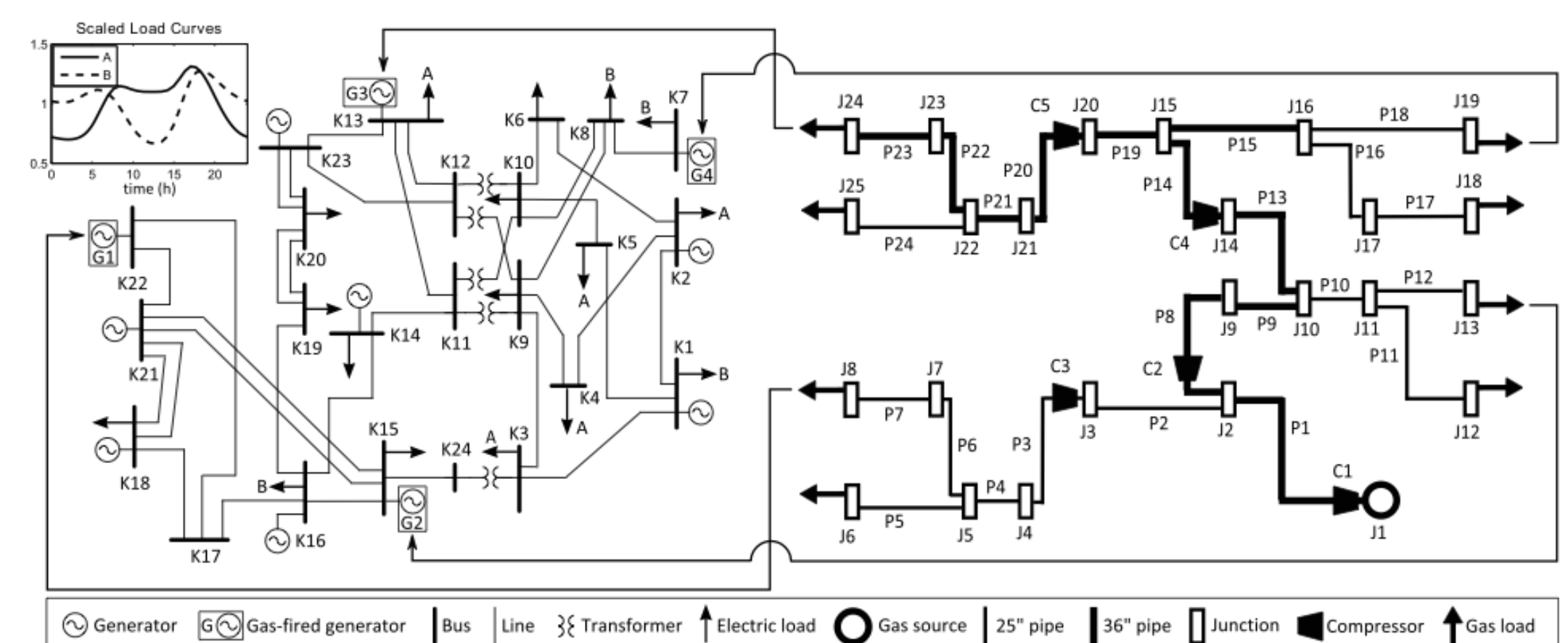
- Infrastructure: fictitious transportation network modeled after the Bay Area
- Power: modified IEEE-9 bus test case



Cost of NE vs Social Optimal solution can be bounded by the Price of Anarchy (PoA), can be calculated in closed form.

- NE exists in the simulated case
- Bi-level optimization does not find the socially optimal solution
- PoA for this case is bounded by 1.46.

GAS PIPELINE SYSTEM COUPLED WITH POWER SYSTEM



^a A. Zlotnik, L. Roald, S. Backhaus, M. Chertkov, and G. Andersson, "Coordinated scheduling for interdependent electric power and natural gas infrastructures," IEEE TPS, 2016.

- Prior study^c showed that when the Max Social Surplus problem was solved separately, then system instability may occur.
- We anticipate that the situation can be exacerbated with competitive gas retailers. \rightarrow Security issue!

FUTURE WORK

- Gas network: do NE points exist?
- Looking at attacks to control system for pipelines or grid that can induce dynamic resonance for pipelines

M. Alizadeh, H.-T. Wai, A. Goldsmith, and A. Scaglione, "Optimal Electricity Pricing for Societal Infrastructure Systems", in 50th HICSS Conference.

M. Alizadeh, H.-T. Wai, A. Goldsmith, and A. Scaglione, "Marginal Charging Station Pricing in an Intelligent Electric Transportation System", accepted by ACC 2017.