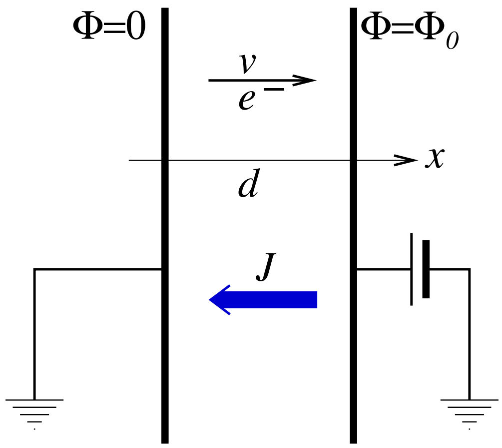
**EM** A vacuum-tube diode is constructed out of two parallel conducting plates that are separated by a uniform spacing $d$ which is much smaller than the dimensions of the plates so that edge effects can be ignored. The cathode at $x = 0$ is grounded, and hence at potential $\Phi(0) = 0$. The anode at $x = d$ is maintained at a fixed positive potential $\Phi(d) = \Phi_0$.




Assume that electrons (mass $m$, charge $q = -e$) are liberated at rest ($v = 0$) at the cathode, and are then accelerated toward the anode where they are absorbed. The charge density $\rho(x)$ and the $x$ component $v(x)$ of the electron velocity do not depend on $y$ or $z$. The system is in a steady state.

a) Show that that the current density $J$ between the plates does not depend on $x$.

b) Use one of Maxwell's equations to show that in the region between the plates

$$\frac{d^2\Phi}{dx^2} = K\Phi(x)^{-1/2},$$

where $K$ is a dimensionful constant depending on $J$, $m$, $e$, and $\epsilon_0$ that you should find.

c) Show that you can integrate the equation in part (b) to obtain

$$\frac{d\Phi}{dx} = K'\Phi(x)^{1/4},$$

where $K'$ is another constant that you should find.

d) Solve the equation in part (c) so as to find the $x$ dependence of $\Phi(x)$, and hence that of $\rho(x)$ and $v(x)$.

e) Find an expression for the magnitude of the current density $J$ in terms of $\Phi_0$, $d$ and other constants.