EM We wish to find the force on a neutral atom in a laser beam whose intensity varies with position. Assume that the (visible $\omega \approx 10^{15}$Hz) light beam can be approximated by a linearly polarized electric field

$$\mathbf{E} = E_0 \mathbf{e}_z \cos(kx - \omega t),$$

and that the magnetic field $\mathbf{B}$ bears the same relation to $\mathbf{E}$ that it would in any plane wave. The amplitude E_0 varies with position on a scale that is long compared to the wavelength of the light.

a) Treat the atom as a dipole of strength $\mathbf{p} = e\mathbf{d}$ in which charges $+e$ and $-e$ are separated by a displacement vector $\mathbf{d}$. Show how that the total force on the atom can be written as,

$$\mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E} + \frac{d\mathbf{p}}{dt} \times \mathbf{B} + \mathbf{F}_{\text{extra}}.$$

Explain why $\mathbf{F}_{\text{extra}}$ is negligible if the atomic speed $\mathbf{v}$ is of order 1m/sec. To find $\mathbf{p}(t)$, use a classical model in which the electron (mass m, charge $-e$) is bound to the heavy ($M \gg m$) nucleus with a spring, and that an atom at $x = 0$ responds to the electric field as a damped oscillator of natural frequency ω_0 and damping time τ:

$$\frac{d^2}{dt^2}\mathbf{d} + \frac{1}{\tau}\frac{d}{dt}\mathbf{d} + \omega_0^2 \mathbf{d} = -\frac{eE_0}{m}\mathbf{e}_z \cos(\omega t).$$

(We are assuming that we can ignore the effect of the magnetic field when computing $\mathbf{p}(t)$.)

b) Show that (ignoring transient effects)

$$\mathbf{p}(t) = \left(a(\omega)\cos(kx - \omega t) + b(\omega)\sin(kx - \omega t)\right) E_0 \mathbf{e}_z,$$

and find the in- and out-of-phase amplitudes $a(\omega)$ and $b(\omega)$.

c) Find the force on the atom averaged over one period of the light wave.

d) If we want the atom to be strongly pulled toward the highest intensity of the laser beam, which should ω be: i) equal to ω_0, ii) close to and greater than ω_0, iii) close to and less than ω_0?

e) In what direction does the $\dot{\mathbf{p}} \times \mathbf{B}$ force act?