EM1 The figure shows an ideal dipole of dipole moment $p$ that lies in the center of a grounded conducting spherical shell of inner radius $R_{1}$ and outer radius $R_{2}$.

a) Obtain a fully explicit expression for $V(r, \theta)$ in the three regions $r \leq R_{1}$, $R_{1}<r<R_{2}$, and $r>R_{2}$.
b) Compute the surface charge density $\sigma(\theta)$ on the inner surface of the grounded conductor.
c) Compute the surface charge density $\sigma(\theta)$ on the outer surface of the grounded conductor
d) Now suppose that in addition to the dipole a point charge $q$ is placed in the center of the sphere. Repeat your calculation of the surface charge distributions on the inner and outer surfaces of the conducting shell.
Hint: Recall that the general axisymmetric solution to Laplace's equation can be written

$$
V(r, \theta)=\sum_{n=0}^{\infty}\left(a_{n} r^{n}+\frac{b_{n}}{r^{n+1}}\right) P_{n}(\cos \theta)
$$

and that $b_{1}=p$ for an isolated dipole. See the formula sheet for explicit expressions for the $P_{n}(\cos \theta)$.

