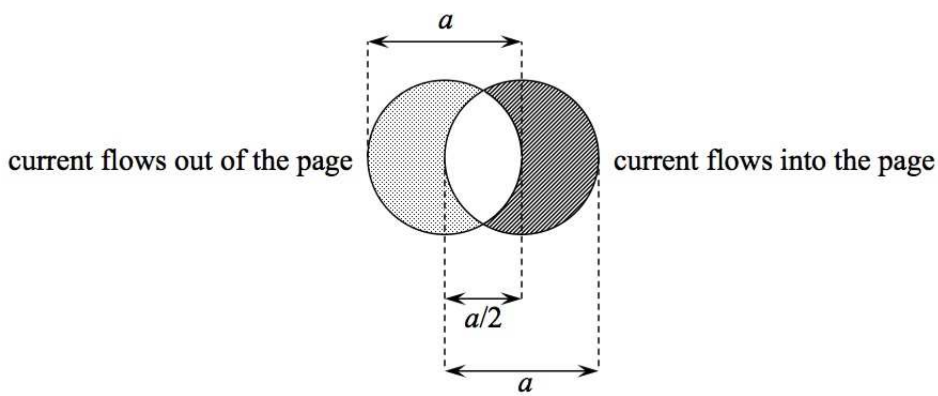
BEM. The dipole magnets that guide the proton beam in a circular accelerator such as the LHC are required to produce a very uniform magnetic field at right-angles to the direction of beam propagation. Suppose that a magnet is built from a long pair of partial cylindrical conductors as shown in cross section in the figure. The region where the geometrical cylinders overlap has no current; it is the vacuum through which the proton beam propagates, and where we wish the magnetic field to be uniform.



a) Begin with the simpler problem of the field produced by a single *complete* conducting cylinder of radius $r = a/2$ that is carrying a uniform current density J directed into the page. Find an expression, in terms of J, a and the permeability of free space μ_0, that gives the magnitude and direction of the field for $r < a/2$, *i.e. within* the current-carrying region.

Now consider the magnet consisting of the two *partial*-cylinder conductors shown in the figure. The left moon-shaped conductor carries a uniform current density $J = 35 \times 10^6 \mathrm{A \cdot m^{-2}}$ directed out of the page. The right moon-shaped conductor carries the same current density, but into the page. The diameter of the cylinders is $a = 0.5$m. The centers of the cylinders are offset horizontally by $a/2$.

b) Find an algebraic expression for the magnitude and direction of the magnetic field in the vacuum between the conductors in terms of J, a, μ_0, and show that the field is indeed uniform.

c) Calculate the numerical value of the magnetic field in Tesla. Recall that $\mu_0 = 4\pi \times 10^{-7} \mathrm{T \cdot m \cdot A^{-1}}$.