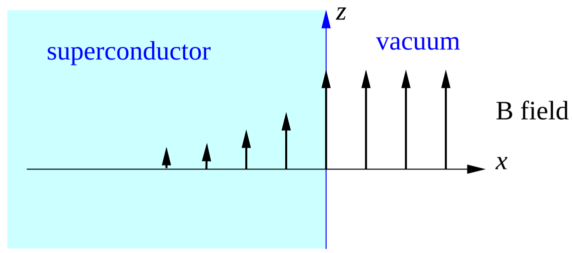
AEM. The Meissner effect is the expulsion of the magnetic field from a superconductor. Fritz and Heinz London introduced an electromagnetic theory to explain this effect.



a) The London brothers assume that the current in the superconductor is carried by a fixed density ρ_0 of particles of charge q and mass m that are moving with a position-dependent velocity $\mathbf{v}$. Express the resulting current density $\mathbf{j}$ in terms of q, ρ_0, and $\mathbf{v}$.

b) The second property assumed by the brothers is that the velocity $\mathbf{v}$ and the magnetic field $\mathbf{B}$ are linked by the condition

$$m \operatorname{curl} \mathbf{v} + q\mathbf{B} = 0.$$

Use Maxwell's equations to show that, in a steady state, the brothers' condition causes the magnetic field $\mathbf{B}$ to obey the equation

$$\nabla^2 \mathbf{B} - \kappa^2 \mathbf{B} = 0,$$

where κ^2 is a constant you should express in terms of ρ_0, q, m, and the permeability of free space μ_0.

c) Consider a superconductor occupying the region $x < 0$ and with a planar boundary in the yz plane. A static magnetic field $\mathbf{B} = (0, 0, B_z)$ is applied parallel to the superconductor in the z direction (see figure). How far into the superconductor does the field penetrate? That is, after what distance has the field diminished to $1/e$ of its strength at the boundary?

d) Find the current $\mathbf{j}$ (a function only of x) that is flowing in the superconductor and screening the magnetic field.

Possibly useful formulæ:

$$\begin{aligned} \operatorname{div}(\mathbf{a} \times \mathbf{b}) &= \mathbf{b} \cdot \operatorname{curl} \mathbf{a} - \mathbf{a} \cdot \operatorname{curl} \mathbf{b}, \quad \operatorname{curl}(\operatorname{curl} \mathbf{A}) = \nabla(\operatorname{div} \mathbf{A}) - \nabla^2 \mathbf{A} \\ \operatorname{curl}(\mathbf{a} \times \mathbf{b}) &= \mathbf{a}(\operatorname{div} \mathbf{b}) - \mathbf{b}(\operatorname{div} \mathbf{a}) + (\mathbf{b} \cdot \nabla)\mathbf{a} - (\mathbf{a} \cdot \nabla)\mathbf{b} \end{aligned}$$