EM

A waveguide is constructed from a rectangular tube of perfectly conducting material. Its hollow core occupies the region $0 < x < a$, $0 < y < b$ Assume that $a > b$. The z axis runs along the tube. The tube is filled with a dielectric with permittivity ϵ and permeability μ. You will investigate the properties of *transverse electric*, or "TE" modes, in which the longitudinal component E_z of the electric field is identically zero. You are therefore seeking solutions to Maxwell's equations of the fom

$$\begin{aligned}\mathbf{E}(x,y,z,t) &= (E_x(x,y), E_y(x,y), 0)\exp\{ik_z z - i\omega t\},\\ \mathbf{B}(x,y,z,t) &= (B_x(x,y), B_y(x,y), B_z(x,y))\exp\{ik_z z - i\omega t\}.\end{aligned}$$

a) What boundary conditions are obeyed by the **E** and **B** fields at the surface of the perfect conductor?

b) It is possible to introduce a scalar function $\psi(x,y)$ in terms of which one (and only one) of the following pairs of equations holds

$$B_x = \frac{\partial\psi}{\partial x}, \quad B_y = \frac{\partial\psi}{\partial y}, \quad \text{(i)}$$
$$E_x = \frac{\partial\psi}{\partial x}, \quad E_y = \frac{\partial\psi}{\partial y}, \quad \text{(ii)}$$

Which pair, (i) or (ii), holds as consequence of Maxwell's equations and the condition $E_z = 0$?

c) Use your result for part (b) to derive the equation obeyed by the scalar function $\psi(x,y)$ in the interior of the waveguide, and also the boundary condition that ψ obeys at surface ($x = 0, a$ or $y = 0, b$) of the perfect conductor.

d) Waves with an angular frequency ω below some cutoff frequency ω_{min} are unable to propagate in the waveguide. Solve your equation from part (c) so as to find the mode with the <u>lowest</u> cutoff frequency and express ω_{min} in terms of a, b, μ and ϵ.

e) Use your result from part (d) to write down the relation between ω and k_z for frequencies above ω_{min}. Show that the phase velocity ω/k_z is greater that the speed of light for all such frequencies. Explain why can we not make use of this fact for faster-than-light communication.