EMB A thin spherical shell of radius R has a uniform surface charge

$$\sigma = Q/4\pi R^2.$$

The shell and charge are rotating slowly (so relativistic effects are negligible) at angular velocity $\boldsymbol{\omega}$.

a) By considering the current caused by the moving charge, compute the magnetic dipole moment of the rotating sphere and show that it is of the form $\boldsymbol{\mu} = C\boldsymbol{\omega}$. You should express the constant C in terms of the total charge Q, the radius R, and perhaps other constants that depend on the system of units you are using. (Hint: you may cite without proof the fact that the moment of intertia of a spherical shell of total mass M is given by $I = 2MR^2/3$.)

It is guessed by a student that the $\mathbf{B}$ field produced by the rotating charge distribution must be of the form

$$\mathbf{B}(\mathbf{r}) = \begin{cases} (\mu_0/4\pi r^3)(3(\mathbf{m}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}), & r > R, \\ A\,\mathbf{m}, & r < R, \end{cases}$$

where A is a constant, $\mathbf{m}$ is some vector parallel to $\boldsymbol{\omega}$, and $\hat{\mathbf{r}} = \mathbf{r}/r$ is the unit vector in the radial direction.

b) Explain what general boundary condition determines the discontinuity in direction and magnitude of a magnetic field across a surface current.

c) For the student's guessed $\mathbf{B}$ field, use your general boundary condition from part (b) to determine first the constant A, and then the surface current that gives rise to her field.

d) Compare your answer to part (c) with the surface current that is actually flowing in the rotating charged shell. Do they agree for some value of $\mathbf{m}$?

e) Compute the field momentum density $\mathbf{E}\times\mathbf{H}/c^2$. The total angular momentum in the electromagnetic field is of the form $I_{\text{field}}\boldsymbol{\omega}$. Is the constant I_{field} positive or negative?