Q3 A charge Q moves on the x axis at constant relativistic velocity $-v\hat{\mathbf{x}}$. At the moment that the charge Q is at the origin, the fields at the point $(x, y, z) = (0, d, 0)$ are

$$\begin{aligned} \mathbf{E} &= \frac{Q}{4\pi\epsilon_0}\frac{\gamma}{d^2}\hat{\mathbf{y}}, \\ \mathbf{B} &= -\frac{Q}{4\pi\epsilon_0}\frac{v}{c^2}\frac{\gamma}{d^2}\hat{\mathbf{z}}, \end{aligned}$$

where

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}.$$

A test charge q is moving parallel to the x axis on the line $y = d$, $z = 0$ at the same constant relativistic speed — but in the *opposite* direction. It reaches the point $(0, d, 0)$ at the same time that charge Q is at the origin.

a) Find the force on q due to Q at the moment that Q is at the origin and q is at $(0, d, 0)$.

b) In the present frame of reference, the fields produced by Q obey the condition $\mathbf{E} \cdot \mathbf{B} = 0$. Will this condition remain true in all inertial frames?

For the remaining problems consider the frame in which the charge q is at rest.

c) Find the speed of the charge Q in the frame in which q is at rest.

d) Find the $\mathbf{E}$ and $\mathbf{B}$ fields at the position of test charge q due to charge Q, at the moment that they have the same x co-ordinate in the frame in which q is at rest. You may either Lorentz transform the given fields, or use your result from part (c).

e) Find the force on q due to Q, at the moment they have the same x co-ordinate in the frame in which q is at rest.

f) Should the forces in parts (a) and (e) be equal? If so, why? If not, why not?