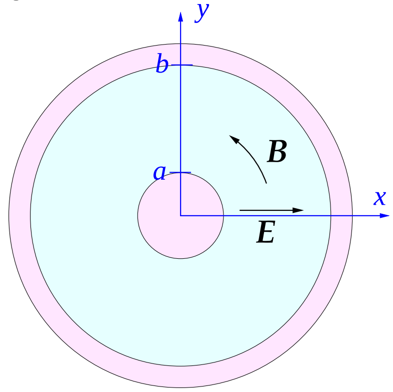
Q2 The figure shows a cross-section of a coaxial cable that is cylindrically symmetric about the z axis. The outer conductor is at radius $r = b$. The inner solid conductor has radius $r = a$. Between the two (perfect) conductors is an insulator possessing dielectric constant ϵ and magnetic permeability μ.



When a signal is propagating, the electric field has radial component

$$E_r(r, \theta, z, t) = \mathcal{E}(t, z)/r, \qquad a < r < b,$$

and the magnetic field has angular component

$$B_\theta(r, \theta, z, t) = \mathcal{B}(t, z)/r, \qquad a < r < b.$$

All other field components are zero, and there is no field in any other region.

a) Use one of Maxwell's equations to find a relation between $\partial\mathcal{B}/\partial t$ and $\partial\mathcal{E}/\partial z$.

b) Use another Maxwell equation to find a relation between $\partial\mathcal{E}/\partial t$ and $\partial\mathcal{B}/\partial z$.

c) Using your answers to parts (a) and (b), find an expression for the phase velocity of electromagnetic waves propagating along the cable.

d) Assume that the current $I(t)$ on the central conductor, and the potential difference $V(t)$ between the two conductors, are of the form

$$I(t) = \mathrm{Re}\left[I_0 e^{i(kz-\omega t)}\right], \quad V(t) = \mathrm{Re}\left[V_0 e^{i(kz-\omega t)}\right].$$

Here I_0 and V_0 can be complex numbers. Use your results from parts (a), (b) and (c) to compute the complex impedance $Z = V_0/I_0$ as a function of ϵ, μ, a and b.