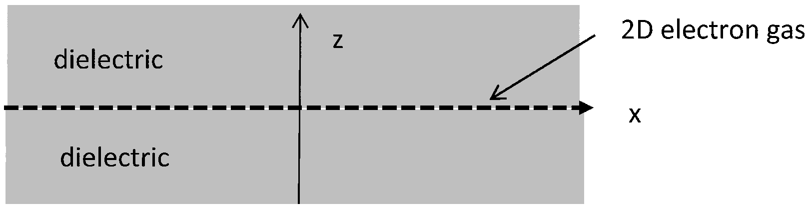
EM



A two dimensional gas of electrons is confined at $z = 0$ in the x-y plane at the interface between two semi-infinite dielectric slabs, as shown above. Each slab has dielectric constant ϵ. A perturbation of the electron charge-density propagates through the electron gas. The total two dimensional charge density is given by $\sigma(x,t) = \sigma_0 + \delta\sigma(x,t)$, where the perturbation $\delta\sigma$ takes the form of a wave

$$\delta\sigma(x,t) = a\exp\{i(kx - \omega t)\}.$$

You may work in either SI or CGS units and assume that:

- Electrons act as classical particles of mass m with local velocity,

$$v(x,t) = v_0 \exp\{i(kx - \omega t)\}.$$

- Magnetic fields are negligible.
- The perturbation is small, *i.e.* $\delta\sigma \ll \sigma_0$.

Now do the following:

a) Use Laplace's equation to find the electrical potential $\phi(x,z,t)$ due to the periodic charge perturbation.

b) From $\phi(x,z,t)$ find the electric field component $E_x(x, z = 0, t)$ parallel to and within the electron gas.

c) Use the linearized charge continuity equation to find a relation between a and v_0.

d) Show that the relation between ω and k for the wave is given by,

$$\omega^2 = \gamma|k|,$$

where you should express the coefficient γ in terms of m, ϵ, σ_0 and the electron charge $q = -e$.