EM An infinitesimally thin sheet of material extends to infinity in the x and y directions. We will apply electric and magnetic fields to the system that do not depend on z, so you may use the three-dimensional Maxwell equations with all derivatives with respect to z set to zero.

a) The *magnetic-flux quantum* Φ_0 is defined by $\Phi_0 = 2\pi\hbar/e$ in SI units ($2\pi\hbar c/e$ in CGS units). Here e is the electron charge. Show that Φ_0 has units of magnetic flux. How strong a magnetic field (in Tesla for SI, in Gauss for CGS) produces a flux Φ_0 through a surface with an area of 1 cm^2?

b) At time $t = 0$ we turn on a space and time dependent electromagnetic vector-potential

$$A_x(x, y, t) = \frac{\Phi_0}{2\pi T}\frac{-yt}{x^2 + y^2}, \quad A_y(x, y, t) = \frac{\Phi_0}{2\pi T}\frac{xt}{x^2 + y^2}.$$

Here T is a constant with units of time, and we are working in a gauge with the scalar potential $V = 0$. Calculate the magnetic flux passing through a disk of radius R centered at the origin. Write the flux in units of Φ_0. Take the limit as $R \to 0$ to determine the flux passing through an infinitesimal disk centered at the origin. HINT: It is easiest to do this using polar coordinates.

c) Use Maxwell's equations to find the electric field as a function of time everywhere in the plane except the origin.

d) Suppose the electric current in the sheet is determined by the electric field as

$$j_x = \frac{ne^2}{2\pi\hbar}E_y, \quad j_y = -\frac{ne^2}{2\pi\hbar}E_x.$$

Here n is an integer, j_x, j_y, are the components of the two-dimensional current density (units of current per length), and E_x, E_y, are the components of the electric field. Use the electric field you derived in part (c) to calculate the total amount of electric charge that has flowed into (or out of) the disk of radius R centered at the origin during the time period from $t = 0$ to $t = T$.

e) Hence find the relation between the number of electrons accumulated in an infinitesimal disk centered at the origin and the amount of flux passing through that disk.