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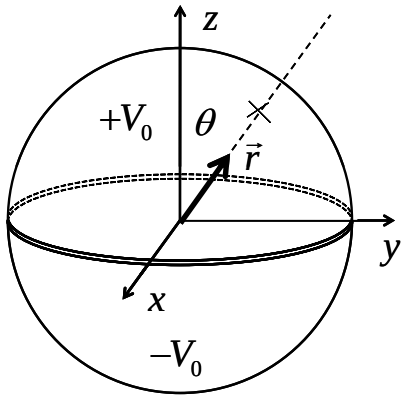
A thin, spherical shell of radius R has a potential of $+V_0$ on the northern hemisphere and a potential of $-V_0$ on the southern hemisphere.



(a) Find the magnitude and direction of the electrical field at the center of the shell by expanding $V(r)$ as:

$$V = \sum_{\ell=0}^{\infty}\left(a_\ell r^\ell + \frac{b_\ell}{r^{\ell+1}} \right) P_\ell(\cos\theta)$$

(b) Show that $V(r)$ only includes contributions from the odd ℓ terms.

(c) Calculate $V(r)$ up to and including the $\ell = 3$ term for $r \ll R$. Use this expression for $V(r)$ to calculate $\vec{E}$ near the origin using Cartesian coordinates and unit vectors. Check your answer by showing that $\vec{\nabla} \bullet \vec{E} = 0$.

(d) Calculate the potential to leading order in $\frac{1}{r}$ in the region $r \gg R$.

Hint $\int_{-1}^{1} P_\ell(x) P_{\ell'}(x) dx = \frac{2}{2\ell+1}\delta_{\ell\ell'}$