## 3



A solenoid of radius $a$ and length $L$ with $a \ll L$ is placed with its axis along the $z$-axis of coordinates, with one end at $z=0$ and the other end at $z=L$. The solenoid is wrapped with $N$ turns of wire, each carrying current $I$. The sense of the current is shown in the figure.
(a) In the approximation in which the magnetic field inside the solenoid is regarded as uniform, and the magnetic field outside the solenoid is neglected, calculate the field (magnitude and direction) inside the solenoid.
(b) There is an additional external uniform magnetic field of magnitude $B_{0}$, directed along the $+x$ axis. Calculate the torque this field exerts on the solenoid. Specify the magnitude and sense of the torque.
(c) The approximation made in (a) can be improved upon by placing fictitious magnetic poles at the ends of the solenoid. The magnetic field due to such a pole of strength $m$ placed at the origin is

$$
\vec{B}=\frac{\mu_{0}}{4 \pi r^{2}} m \hat{r}
$$

The poles are to be placed at $x=y=z=0$ and $x=y=0, z=L$. In this improved model, the net magnetic field is the sum of the solenoid field (assumed to be zero outside the solenoid) and the monopole fields. Use the result of (a) to find the flux inside the solenoid, and then go on to determine the strength of the pole at $x=y=0, z=L$ by demanding that the flux inside the solenoid equal the flux which emerges from this monopole into the region exterior to the solenoid. Likewise determine the strength of the pole at $x=y=0, z=0$ by demanding that the flux headed toward this pole from the region exterior to the solenoid equal the flux inside the solenoid. Sketch the resulting distribution of field lines inside and outside the solenoid. Now combine the result of (a) and the contribution of the magnetic poles to get an improved value for the net magnetic field at two points: (i) at the center of the solenoid, $x=0, y=0, z=L / 2$ and (ii) at a point just outside the solenoid, $x=2 a, y=0, z=L / 2$.
(d) Using the formula $\vec{F}=m \vec{B}$ for the force on a magnetic pole in a magnetic field, calculate the torque exerted on the system of poles found in (c), exerted by the external field $B_{0}$. Compare your answer to that found in (b).

