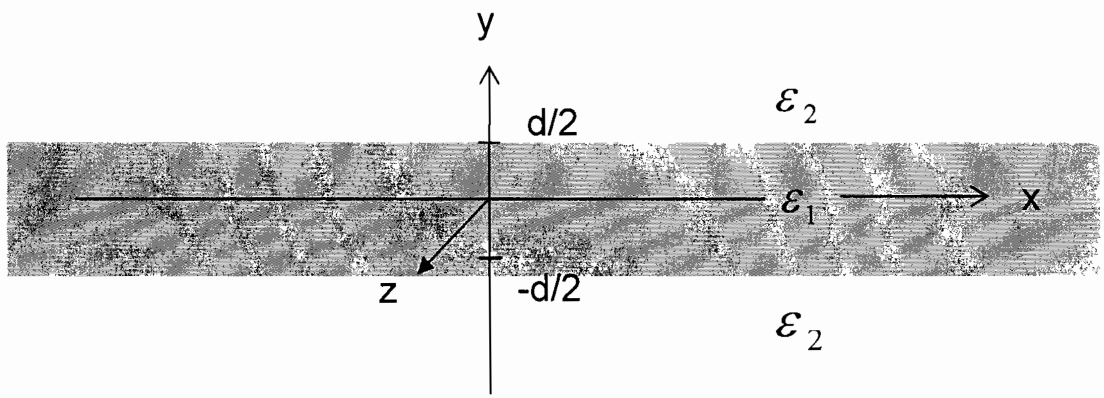
A dielectric waveguide consists of a slab with dielectric constant ε_1 embedded in an infinite medium with dielectric constant $\varepsilon_2 < \varepsilon_1$. The indices of refraction are n_1 and n_2 respectively. The slab extends infinitely far in the x and z directions and has thickness d in the y-direction. For modes that *propagate* in the z direction and are *localized* to the slab region the electric field takes the form, $\vec{E} = \hat{x} E_x(y) \exp(i(kz - \omega t))$. Inside the slab, $E_x \sim \cos q_1 y, \sin q_1 y$ while outside the slab $E_x \sim \exp(\pm q_2 y)$.



(a) Use geometrical optics to show that propagating modes satisfy,

$$q_1^2 < \left(\frac{\omega}{c}\right)^2 \left(n_1^2 - n_2^2\right)$$

(b) Use Maxwell's equations to show that the electric field in each region (i = 1, 2) satisfies,

$$\frac{\partial^2 E_x}{\partial y^2} = \left(k_z^2 - \left(\frac{\omega n_i}{c}\right)^2\right) E_x$$

(c) Show that the magnetic field has both transverse and longitudinal components.

(d) Show that the propagating modes must satisfy continuity of both E_x and $\partial E_x / \partial y$ at y = ± d/2.

(e) Make a rough sketch of E_x in regions 1 and 2 and explain why the lowest frequency mode is even ($E_x(y) \sim \cos q_1 y$) rather than odd ($E_x(y) \sim \sin q_1 y$)

(f) Using the boundary conditions and the wave equation, shown that the even modes satisfy,

$$\tan(q_1 d/2) = \frac{q_2}{q_1} \quad , \quad q_2 = \sqrt{\left(\frac{\omega}{c}\right)^2 (n_1^2 - n_2^2) - q_1^2}$$

(g) Sketch q_2/q_1 and $\tan(q_1 d/2)$ versus q_1 and show that there is at least one propagating mode so long as $n_1 > n_2$.