

A wire loop (loop 1 in the figure) of radius  $a$ , centered at the origin of a cylindrical  $(\mathbf{r}, \mathbf{f}, z)$  coordinate system, lies in the  $z = 0$  plane and has a current  $I_1 = I_0 \cos(\mathbf{w}t)$  flowing around it. A second loop (loop 2 in the figure), of radius  $b$ , lies in the  $z = z_0$  plane with its center on the  $z$ -axis. Assume that  $z_0 \gg a$  and  $z_0 \gg b$ . (The figure is not to scale.) Also, ignore radiation effects.

- (a) Consider the magnetic field produced at position  $\mathbf{r} = b$  and  $z = z_0$  (i.e., on loop 2) by the current  $I_1$ . The field is proportional to  $I_1$ :

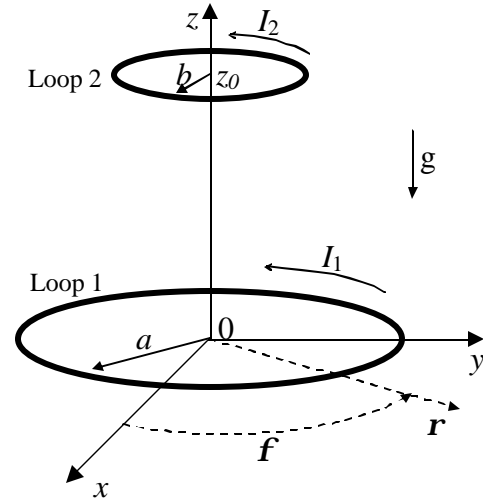
$$B_r = a_r I_1 z_0^a$$

$$B_f = a_f I_1 z_0^b,$$

$$B_z = a_z I_1 z_0^g$$

where  $a_r$ ,  $a_f$ , and  $a_z$  are not functions of  $I_1$  or  $z_0$ .

- i. Show that  $a_f = 0$ .
- ii. What are the values of  $a$  and  $g$ ?



In case you could not solve part (a), you may express your answers to parts (b), (c), and (d) in terms of  $a$  and  $g$ , as well as the other quantities defined in the problem.

- (b) Assume that loop 2 has self inductance,  $L$ , and negligible resistance. Determine  $I_2$ , the current induced in loop 2 as a function of time. Take positive current to be in the direction of increasing  $\mathbf{f}$ , as shown in the figure.
- (c) Calculate the  $z$ -component of the force on the second loop as a function of time.
- (d) Assume that  $\mathbf{w}$  is sufficiently large that the height of loop 2 (which has mass,  $m$ ) is determined only by the time averaged force. Determine the equilibrium height,  $z_0$ . Gravity,  $\bar{g}$ , points in the  $-z$  direction.
- (e) Calculate  $a_r$  and  $a_z$  in terms of  $a$ ,  $b$ ,  $z_0$ , and constants. Keep only the largest nonvanishing terms in  $a/z_0$  and  $b/z_0$ .