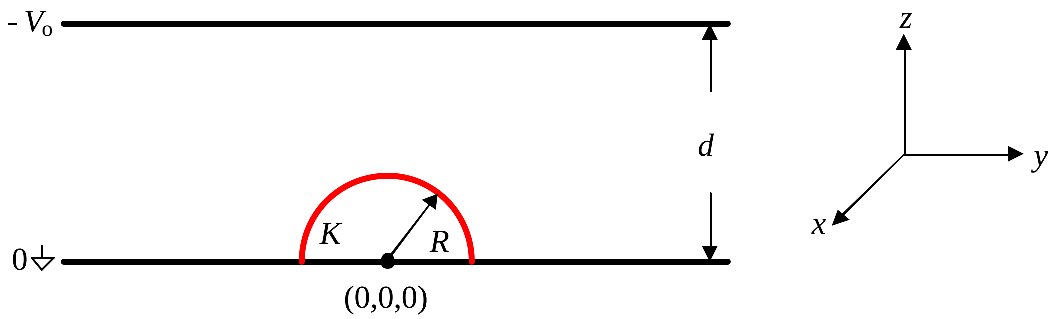

An initially uniform electric field, $\vec{E}(\vec{r}) = E_o\hat{z}$ exists in between two infinitely large, parallel conducting plates, separated by a small vertical distance, d. The lower plate is at ground potential (0 volts) and the upper plate is held at a constant potential $-V_o$. A hemi-spherical dielectric, with dielectric constant $K \equiv e/e_o > 1$ and radius $R << d$ is placed on the lower plate with its center at the origin of coordinates of the parallel-plate capacitor as shown in the above figure.

(a) State the boundary conditions for $\vec{E}(\vec{r})$ and $\vec{D}(\vec{r})$ at the surface of the hemi-spherical dielectric.

(b) Calculate the potential inside ($\Phi_{in}(\vec{r})$) and outside ($\Phi_{out}(\vec{r})$) the hemi-spherical dielectric.

Hint: You may use the following formula (without derivation):

$$\Phi(\vec{r}) = \sum_{\ell=0}^{\infty}\left(a_\ell r^\ell + b_\ell \frac{1}{r^{\ell+1}}\right)P_\ell(\cos q)$$

where r and q are spherical polar coordinates.

(c) Calculate the electric field inside ($\vec{E}_{in}(\vec{r})$) and outside ($\vec{E}_{out}(\vec{r})$) the hemi-spherical dielectric.

(d) Make a detailed, careful sketch of the electric field lines and the equipotentials (lines of constant potential) between the parallel plates, and also inside and outside the hemi-spherical dielectric. Draw the electric field lines as solid lines and the equipotentials as dashed lines.

(e) Calculate the bound charge density, s_{bound} on the upper surface of the hemi-spherical dielectric. Express your answer in terms of $E_0 \equiv V_0/d$, K and q.