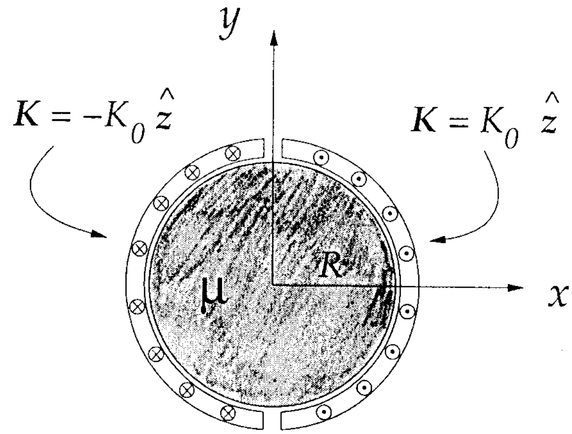
The figure below shows an unusual cable: the conducting surface of the cable is split into two halves by an insulating gap of negligible thickness. The right-hand side (at $x > 0$) carries a uniform surface current density $\mathbf{K} = K_0\,\hat{\mathbf{z}}$, while the left-hand side carries the same current in the opposite direction: $\mathbf{K} = -K_0\,\hat{\mathbf{z}}$. The cable runs along the z axis, has circular cross-section of radius R, and may be considered to be of infinite length. Finally, the core of the cable is filled with a material of magnetic permeability μ.



(a) Since the free volume current density $\mathbf{J}_f = 0$ everywhere, $\nabla \times \mathbf{H} = 0$ and so we can define a magnetic scalar potential, V^*, such that $\mathbf{H} = \nabla V^*$. Further, Maxwell's equations tell us that $\nabla^2 V^* = 0$. Write down the *general form* of the solution for the field $\mathbf{H}(\mathbf{r})$ in cylindrical coordinates, both inside and outside the cable (i.e. your expression will involve arbitrary constants), and state explicitly the *boundary conditions* that must be satisified.

(b) Show that the magnetic fields $\mathbf{B}^{\rm EXT}(\mathbf{r})$ (outside the cable) and $\mathbf{B}^{\rm INT}(\mathbf{r})$ (inside the cable) have the following form:

$$\mathbf{B}^{\rm EXT} = A^{\rm EXT} \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{R}{r}\right)^{n+1} \frac{(-1)^{(n-1)/2}}{n} (-\,\hat{\mathbf{r}} \sin n\phi + \hat{\boldsymbol{\phi}} \cos n\phi)$$

$$\mathbf{B}^{\rm INT} = A^{\rm INT} \sum_{n=1,3,5,\ldots}^{\infty} \left(\frac{r}{R}\right)^{n-1} \frac{(-1)^{(n-1)/2}}{n} (-\,\hat{\mathbf{r}} \sin n\phi - \hat{\boldsymbol{\phi}} \cos n\phi)$$

where and r and ϕ are the usual radial and azimuthal cylindrical coordinates. Also determine the values of the constants $A^{\rm EXT}$ and $A^{\rm INT}$ in terms of the given parameters of the problem and physical constants.

(c) In the far-field limit (i.e. at distances far away from the cable), the leading term in the magnetic field has this form:

$$\mathbf{B} \rightarrow \frac{C}{r^2}(-\hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi),$$

where C is a constant. Not surprisingly, this expression is *also* the dominant far-field behaviour of the magnetic field due to two infinite wires, carrying current I in opposite directions (see figure at right). Find the constant C in terms of the current I and the spacing d between the wires.