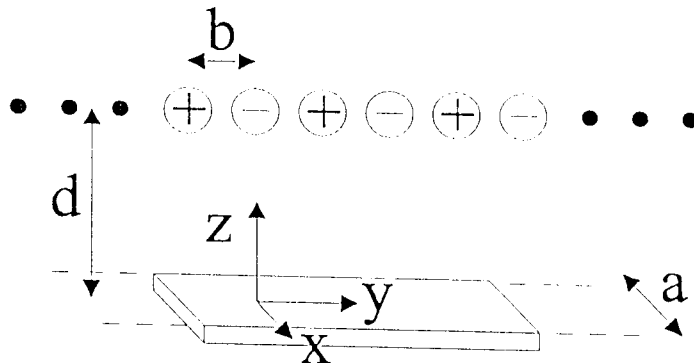


The purpose of this problem is to model the force law between a one-dimensional ionic crystal consisting of an infinite line of point charges of alternating charge $\pm Q$ and an infinite strip of conductor, as shown in the diagram. The calculation might represent the interaction between a polymer and a conductor.



The spacing between the charges is b , and their distance from the conductor is d . The strip of conductor is given a width $a \ll d$.

(a) Assume that the crystal induces a surface charge along the conductor of the form $\sigma(y) = \sigma_0 \cos(\frac{\pi y}{b})$, which is a good approximation. Determine the electrostatic potential of this distribution at the position of one of the ions and hence the potential energy *per charge* of the configuration. You will find the following standard integral useful:

$$\int_0^{\infty} \frac{\cos(\alpha y)}{\sqrt{y^2 + \beta^2}} dy = K_0(\beta\alpha) \approx \frac{\pi}{\sqrt{2\pi\beta\alpha}} \exp(-\beta\alpha) \quad (1)$$

(b) Estimate the energy of the electric field produced by the induced charge $\sigma(y)$. Use the following crude approximation: determine the energy in this electric field in the volume near the conductor, assuming the field is constant in both z and x , and exists only within $0 < z < d$ and $-\frac{a}{2} < x < \frac{a}{2}$. This energy should be stated *per charge* of the lattice.

(c) Obtain the magnitude of the induced charge, the value of σ_0 that minimizes the total electrostatic energy (per ion), which is the sum of the two previous quantities.

(d) Hence obtain an expression for the (attractive) force per charge between the lattice and the conductor as a function of the distance of separation d .