Consider two nearly concentric conducting spherical shells. They have radii a and b, as shown. Their centers are misaligned by a small distance, δ. Charge $+Q$ is placed on the outer sphere and $-Q$ on the inner.

a. Let $\delta = 0$. Calculate the potential $V(r,\theta,\phi)$ in the three regions (I, II, and III) described below:
For convenience, pick $V = 0$ at $r = 0$.

I. Inside the outer surface of the small sphere $(r < a)$,
II. Between the spheres $(a < r < b)$,
III. Outside the inner surface of the large sphere $(r > b)$.

Suppose the large sphere is offset by a small distance, δ, along the z axis, as shown in the figure. The small sphere remains centered at the origin.

b. Calculate the electric dipole moment of the system. (It is not $Q\delta$.)

c. To first order in δ, the inner surface of the outer sphere is described by the equation, $r = b + \delta\cos\theta$. The function that describes the potential in region II must satisfy, at $r = a$ and $r = b$, boundary conditions of the form $V = v_o + v_1\cos\theta$. Calculate v_{oa}, v_{1a}, v_{ob}, and v_{1b}.

d. Calculate, to first order in δ, the potential $V(r,\theta,\phi)$ in the three regions described in part (a). If you were unable to determine v_{oa}, v_{1a}, v_{ob}, and v_{1b} in part (c), leave them as symbols in your answer.