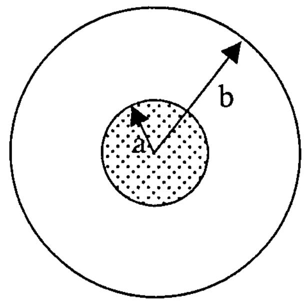
EMSpring 02A

A coaxial cable (transmission line) consists of concentric cylindrical conductors with vacuum in between. The inner conductor has radius a and the outer conductor has radius b as shown in the figure. Assume that both cylinders are perfect conductors.



a) Find the capacitance per unit length C_ℓ.

b) Find the self-inductance per unit length L_ℓ and show that the product $C_\ell L_\ell$ is a universal constant.

In parts c)-e) we will consider the propagation of transverse electromagnetic (TEM) waves of a single frequency, ω, on the cable. TEM waves are characterized by $\vec{E} = \sqrt{\mu_0/\varepsilon_0}\, \hat{z} \times \vec{H}$, where z is the propagation direction, $\vec{E}$ is the electric field, $\vec{H}$ is the magnetic field, μ_0 is the permeability of the vacuum and ε_0 is the permittivity of the vacuum.

c) The characteristic impedance of the cable is defined as the ratio of the voltage across the two conductors at a given point to the current flowing at that same point for an infinitely long cable with no reflected wave. Calculate the characteristic impedance of the cable in terms of a and b and fundamental constants.

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d) Find a pair of coupled differential equations relating the voltage and current as a function of z (the position along the cable) in terms of the capacitance and inductance per unit length of the cable and the frequency ω.

e) Now suppose that the cable is terminated at the far end by a resistive impedence Z. Calculate the reflection coefficient, defined as the amplitude of the reflected voltage to the incident voltage. Find the value of Z such that the reflection coefficient vanishes. Express this special value of Z in terms of the capacitance and inductance per unit length.