

## EM FALL 01 A

An infinitely thin, perfectly conducting sheet lies in the plane  $z = 0$  of an otherwise empty space.

- a) A current is introduced into the conductor so that the  $z$ -component of the  $\mathbf{B}$  field is

$$B_z = B_{z0} \cos(kx)$$

just above and below the sheet; here  $k > 0$ . Find  $\mathbf{B}$  at all points outside the sheet; you may assume that  $B \rightarrow 0$  as  $z \rightarrow \infty$ . Hint: recall that a vector field  $\mathbf{v}$  satisfying  $\nabla \times \mathbf{v} = 0$  may be rewritten in terms of a "potential"  $\psi$  ( $\mathbf{v} = \nabla \psi$ ) satisfying  $\nabla^2 \psi = 0$ .

- b) Sketch the field lines in the  $x$ - $z$  plane, making sure you indicate their direction.

If you were unable to solve part a), in the following write the formulae in terms of the unknown function  $B(x, y, z)$ .

- c) How much work, per unit area in the  $x$ - $y$  plane, was required to produce this electromagnetic field?

- d) Find the surface current density in the sheet.

- e) Now suppose that the conducting sheet is embedded in a medium with constant relative magnetic permeability  $\mu/\mu_0$ , and that currents in the sheet produce a field

$$B_z = B_{z0} \cos(kx)$$

Just above and below the sheet. What is the new surface current density in the sheet?