

EM Spring 01A

A dilute gas has N atoms per unit volume, each with one bound electron of mass m which interacts with an electromagnetic field. Assume that the electron executes simple harmonic motion with natural frequency ω_0 about the atom's center of mass,

$$\frac{d^2x}{dt^2} + \omega_0^2 x = \frac{qE(t)}{m}$$

where $x(t)$ is the electron's position, $E(t)$ is the electric field acting it and q is its charge.

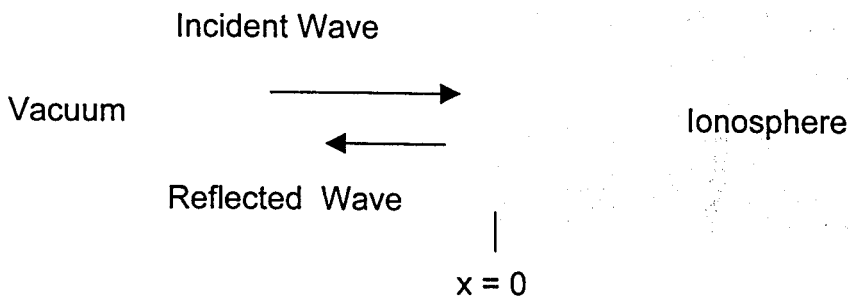
(a) Find a general expression for the dielectric constant $\epsilon(\omega)$ and the index of refraction $n(\omega)$ as a function of frequency.

(b) Find an expression for $n(\omega)$ for radio waves passing through an inert gas such as helium. What are physically reasonable estimates for ω and ω_0 ?

Consider now a dilute plasma consisting of N singly ionized atoms and N electrons per unit volume. Assume there are no static magnetic fields present. This is a model for the ionosphere.

(c) First ignore the ions and find $\epsilon(\omega)$ and $n(\omega)$ for the free electrons in the ionosphere. Find an expression for the plasma frequency, ω_P , below which $\epsilon(\omega) < 0$.

(d) Show that upon entering the ionosphere from a vacuum, the amplitude of a wave with frequency $\omega < \omega_P$ decays as $\exp(-x/L)$ and give an expression for L as a function of frequency. What is the ratio of the incident to reflected power for this wave?



(e) Over what range of frequencies can you safely ignore the response of the ions?