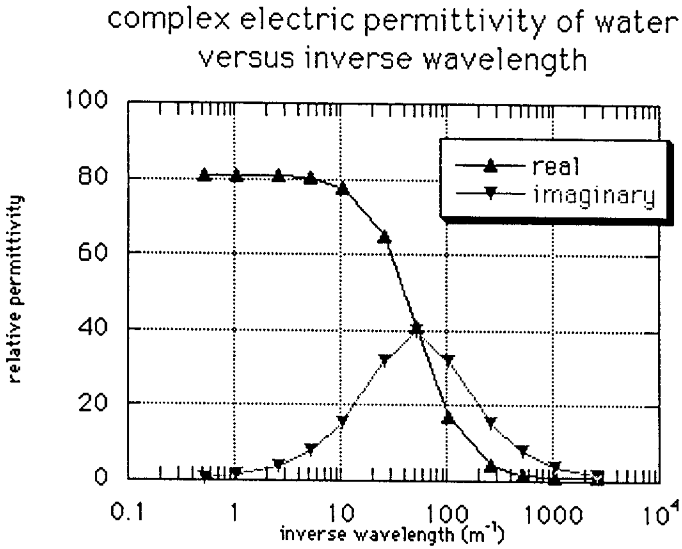
EM 7a1100A

At high enough frequency pure water behaves as a non-conducting dielectric where the electric permittivity is ε and the magnetic permeability is μ. The relative electric permittivity, $\varepsilon_r = \varepsilon / \varepsilon_o$, is complex, i.e. $\varepsilon_r = \varepsilon_{real} + i\, \varepsilon_{imag}$, and the relative magnetic permeability, $\mu_r = \mu / \mu_o = 1$. ε_o and μ_o are the electric permittivity and magnetic permeability of vacuum. The graph below shows ε_{real} and ε_{imag} as functions of inverse wavelength, λ^{-1}. As is customary in the presentation of such data, the wavelength displayed is the wavelength in vacuum.



A) For a transverse electromagnetic plane wave in vacuum, state the relation between the inverse wavelength in vacuum, λ^{-1}, and ω, the angular frequency.

[continued on next page]

For parts B through E below consider a transverse electromagnetic plane wave in water with angular frequency ω, propagating in the z direction. Each component of the electric field is proportional to $\exp(i(kz - \omega t))$, where the wave number, k, is complex, i.e. $k = k_{real} + i\,k_{imag}$.

B) Find an expression for the phase velocity, and find an expression for the distance over which the electric field drops by a factor of 1/e. This distance is the attenuation length.

C) Express ε_{real} and ε_{imag} in terms of k_{real}, k_{imag}, ω and c, the speed of light in vacuum.

D) Find the numerical value of the phase velocity in the low frequency limit.

E) From the graph $\varepsilon_{real} = \varepsilon_{imag} = 40$ at $\lambda^{-1} = 50\ m^{-1}$. At this wavelength ε_{imag} is a maximum. Find the numerical value of the attenuation length at this wavelength.