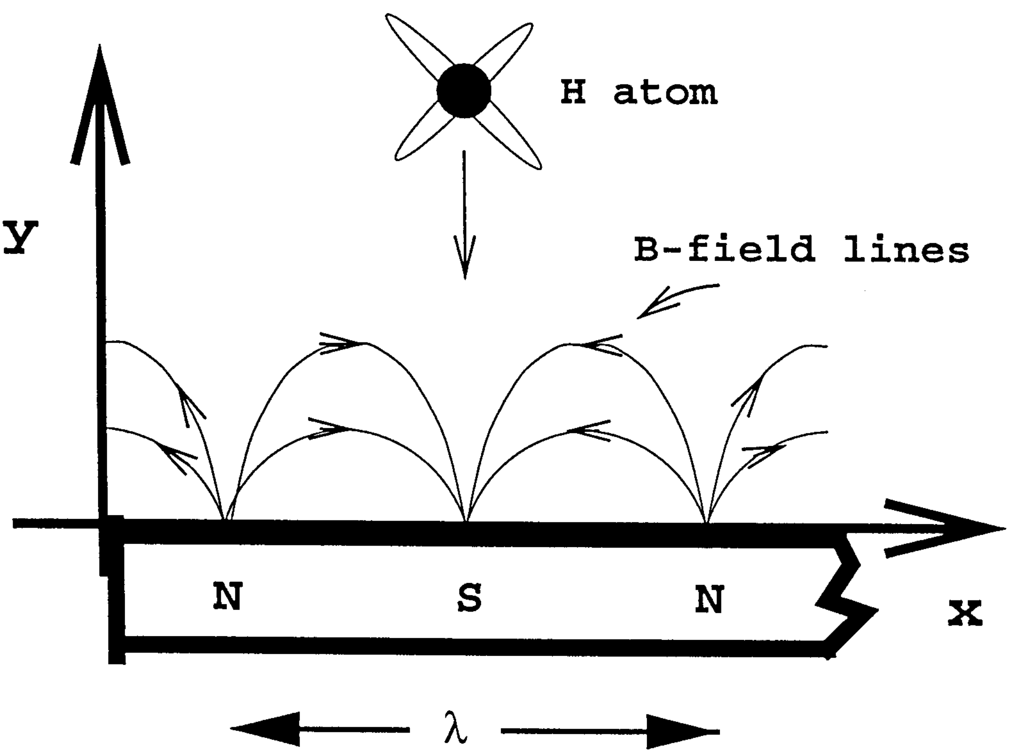
EM 7a 1/99A

This problem illustrates the concept of magetic mirrors for neutral atoms. Consider an infinite sheet of magnetic material placed in the x-z plane in the form of a magnetic recording tape. The tape has a nonuniform periodic magnetization $\mathbf{M} = \hat{\mathbf{x}} cos(2\pi x/\lambda)$, where $\lambda/2$ is the distance between the north (N) and south (S) poles of the magnetization. The region outside the tape is a vacuum. Assume that there are no real currents or time varying fields anywhwere in space.

Assume that the top surface of the tape is at $y = 0$, and that the B-field at the top is given by $\mathbf{B} = B_\circ[\hat{\mathbf{y}} sin(2\pi x/\lambda) - \hat{\mathbf{x}} cos(2\pi x/\lambda)]$. State the units that you are using to solve this problem.



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(a) Show that the **H** field in the vacuum region above the tape may be determined from a magnetic scalar potential $\Phi_m(x, y)$, such that $\mathbf{H} = -\nabla\Phi_m$ in analogy with electrostatics. Show that Φ_m satisfies a two-dimensional Laplace's equation.

(b) Determine the most general solution for $\Phi_m(x, y)$ by solving Laplace's equation.

(c) State the boundary conditions on **B** and **H**, at $y = 0$ and $y = \infty$, that are needed to evaluate **B** in the vacuum region.

(d) Use the boundary conditions to evaluate **B** in the vacuum.

(e) An ultra-cold hydrogen atom of magnetic moment $-\mu\hat{\mathbf{y}}$ approaches the top surface of the tape from above and is traveling initially in the $-\hat{\mathbf{y}}$ direction along the line $x = 3\lambda/4$. Determine the force exerted by the tape on the atom as a function of distance from the tape. Does the tape attract or repel this atom ?