EM Fall98B

An insulating sphere of radius R contains a fixed spherically symmetric distribution of electric charge with density $\rho(r)$. The form of the function $\rho(r)$ as a function of radius r is not specified. The sphere rotates rigidly with angular frequency vector $\mathbf{\Omega} = \Omega\hat{z},\ \Omega > 0$.

A) Give an expression for the electrostatic potential $\phi(\mathbf{r})$ at any point $\mathbf{r}$ in terms of an integral involving $\rho(r)$. The potential is zero at infinity.

B) Give an expression for the current density vector $\mathbf{j}(\mathbf{r})$ in terms of the quantities given, at any point inside the sphere.

C) Give an expression for the magnetic field vector $\mathbf{B}(\mathbf{r})$ at any point $\mathbf{r}$ in space in terms of an integral over the current density found in part (B).

D) Find a simple expression for the ratio $\dfrac{|\mathbf{B}(0)|}{\phi(0)}$ in terms only of Ω and fundamental constants.

E) Give the direction of $\mathbf{B}(0)$ if the charge density is positive everywhere.

HINT: You may find it convenient to use the vector relation

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}$$