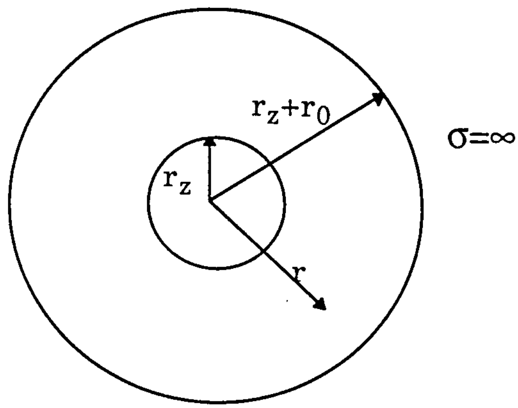
EMSpring 98 B

The atmosphere on the distant planet Zardov can be modeled as a leaky spherical capacitor. In a coordinate system with the origin at the center of Zardov, the conductivity of its atmosphere can be approximated by

$$\sigma = (3 \times 10^{-14} + 5 \times 10^{-15}\,(r-r_z)^2)\ \text{mho}/m \qquad r-r_z < r_0$$

$$\sigma = \infty \qquad r-r_z \geq r_0$$

where $r_z = 10^6$ meters is the radius of Zardov. Assume that at Zardov's surface there exists an electric field of 100 V/*m*, pointed radially inward toward the center of Zardov. For parts (a) and (b) of this problem, assume that the charge on the capacitor is replenished as quickly as it leaks off, so there is a steady state situation. (If you prefer to work in Gaussian units, the dimensions of conductivity are sec^{-1}, and 1 mho/*m* = $9 \times 10^9\ \text{sec}^{-1}$.)



(a) Find the radial component of the current density as a function of r, and evaluate any constants.

(b) Find the charge density in the atmosphere as a function of r.

(c) Because the conductivity σ is discontinuous at $r = r_z+r_0$ there is a very thin shell of charge at this radius. Find the total charge in the atmosphere, including this shell of charge.

(d) If the charge on Zardov were not replenished by, for example, lightning storms, it would be neutralized by the current flow. Assuming that the conductivity is a constant given by its value at Zardov's surface ($\sigma = 3 \times 10^{-14}$ mho/*m*), calculate the time constant for this discharge.