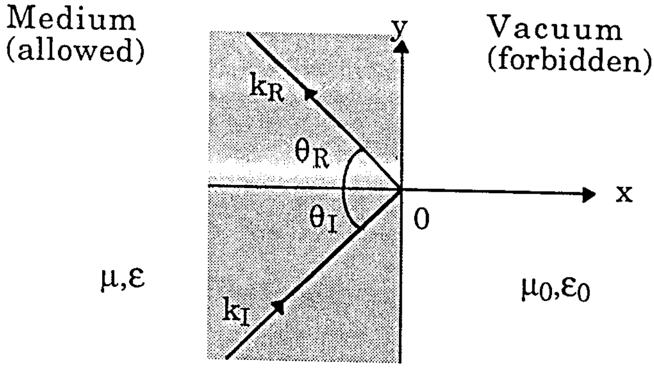

In the above figure, an incident plane wave travelling in a nonconductive medium of permittivity ε and permeability μ is totally reflected at the interface with vacuum at $x = 0$. The incident and the reflected wave vectors lie in the xy-plane,

$$\vec{k}_I = (k_I \cos\theta_I, k_I \sin\theta_I, 0)$$
$$\vec{k}_R = (-k_R \cos\theta_R, k_R \sin\theta_R, 0)$$

Consider the case that the polarizations described by the E-fields of the incident wave, E_I, the reflected wave, E_R, and the wave in the forbidden regions, E_T, are all in the z-direction,

$$\vec{E}_I = (E_I)_0\, e^{i\vec{k}_I\cdot\vec{r}-i\omega t}\,\hat{z}$$

$$\vec{E}_R = (E_R)_0\, e^{i\vec{k}_R\cdot\vec{r}-i\omega t}\,\hat{z}$$

and

$$\vec{E}_T = (E_T)_0\, e^{-\kappa x+ik_T y-i\omega t}\,\hat{z}$$

In the forbidden region ($x > 0$), the E-field is a wave that attenuates in the x-direction, and propagates in the y-direction.

(a) Write down (but you need not derive) the partial differential equation satisfied by the $\vec{E}$ field in the allowed ($x < 0$) and forbidden ($x > 0$) regions, in terms of $\vec{E}$, x, t, v and c. Here

$$v = 1/\sqrt{\varepsilon\mu} \qquad c = 1/\sqrt{\varepsilon_0\mu_0}$$

are the speed of light in the medium and vacuum respectively.

(Cont.)

(b) Hence, show that

$$\omega^2 = k_I^2 v^2 = k_R^2 v^2$$

and

$$\omega^2 = c^2(k_T^2 - \kappa^2)$$

(c) By matching the y-dependence of the three waves, obtain κ and k_T as functions of k_I, θ_I, and the index of refraction $n \equiv c/v$. Find the restriction on θ_I and n such that real κ and k_T solutions exist.

(d) Obtain $\vec{B}_T$ in the forbidden region ($x > 0$). Express your answer in terms of κ, k_T, ω, and $(\vec{E}_T)_0$.

(e) Obtain $\vec{B}_I$ and $\vec{B}_R$ in the allowed region ($x < 0$) in terms of k_I, θ_I, $(E_I)_0$, and $(E_R)_0$.

(f) State the boundary conditions at the interface $x = 0$ that $\vec{E}$ and $\vec{B}$ must satisfy.