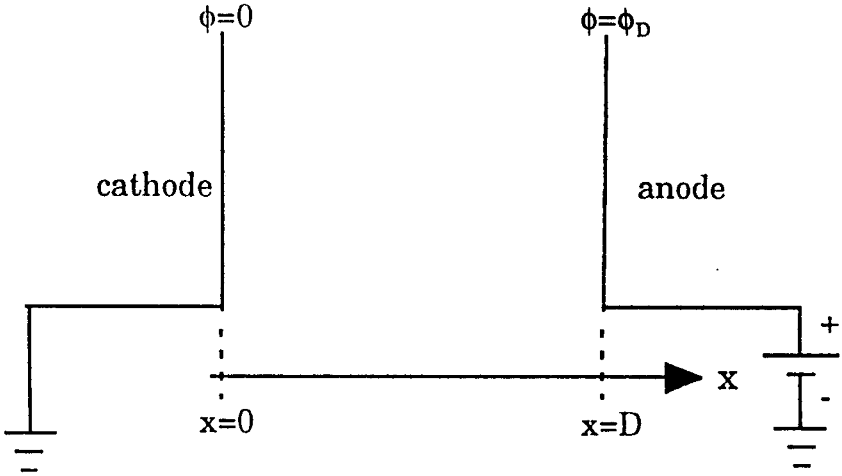
EMSpring96A

The current-voltage relation of a space-charge limited, thermionic diode can be obtained from the following idealized model. Two infinite parallel conducting plates in vacuum are separated by a distance D. The cathode is grounded and is an inexhaustible source of low energy electrons of charge -e. The anode is at positive potential ϕ_D. Let the potential, the electron number density, and the electron velocity at x, $0 \le x \le D$, be denoted by $\phi(x)$, $n(x)$, and $v(x)$, respectively. In steady state conditions these quantities do not depend on time.



(a) Neglecting the small initial velocity of the electrons, find an expression for the electron velocity, $v(x)$, in terms of the potential, $\phi(x)$, and numerical and physical constants.

(b) Find an expression for the current density, $j(x)$, in terms of the quantities defined above, and show that the current density is independent of x.

(c) Starting with Poisson's equation, $\nabla^2\phi(x) = 4\pi e\, n(x)$, and the results of parts (a) and (b), find a differential equation involving only $\phi(x)$, $j(x)$, and numerical and physical constants.

(d) Perform two integrations on the differential equation of part (c) to obtain an equation for $\phi(x)$. For the limiting case of a space-charge limited diode, the boundary conditions at $x = 0$ are $\phi = 0$ and $d\phi/dx = 0$. The relation $\frac{d}{dx}\left(\frac{d\phi}{dx}\right)^2 = 2\left(\frac{d\phi}{dx}\right)\left(\frac{d^2\phi}{dx^2}\right)$ is useful.

(e) Evaluating the solution to part (d) at $x = D$, find an expression for the current density, $j(x)$, in terms of ϕ_D, D, and numerical and physical constants.