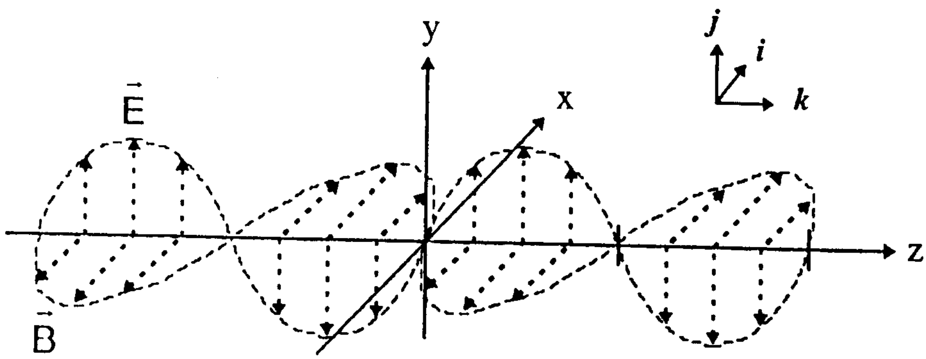
EM Fall95B

In a certain region of space the density of free electrons is ρ. An electromagnetic plane wave with angular frequency ω is incident on this region. You can assume the induced motion of the electrons is such that their velocity is much smaller than the speed of light.

a) Derive an expression for the index of refraction n of this charge distribution as experienced by the plane wave. Your answer should be in terms of ρ and ω, (described above), the electronic charge e and mass m, and the speed of light c.

b) Show that the wavelength λ of light propagating in this region is related to its angular frequency ω by $\lambda = 2\pi c / \sqrt{\omega^2 - \omega_p^2}$ where $\omega_p^2 = 4\pi e^2 \rho / m$. Describe qualitatively what happens when light of frequency $\omega < \omega_p$ is incident on the region.

Now suppose that a plane-wave having frequency ω interacts with a single free electron. With reference to the diagram below, assume the electron's motion is centered on the origin, the wave is propagating along the **z-axis**, and the electric and magnetic fields of the wave at the location of the electron are given by $\mathbf{E} = E_0 \sin(\omega t)\,\mathbf{j}$ and $\mathbf{B} = -B_0 \sin(\omega t)\,\mathbf{i}$ (see picture below). Assume that the speed of the electron is much smaller than the speed of light at all times (in other words, the total force acting on the electron in the **j** direction is dominated by the electric field).



c) By solving for the motion of the electron, show that it behaves like a superposition of two oscillating dipoles, one having frequency ω, and the other having frequency 2ω.