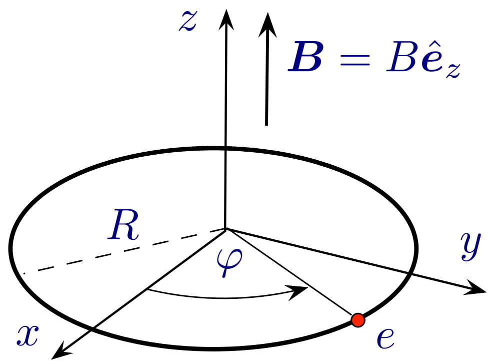
QM A particle of charge e and mass M moves on a frictionless ring in the xy plane in the presence of a magnetic field $\mathbf{B} = B\mathbf{e}_z$. The quantum Hamiltonian is

$$\hat{H} = \frac{1}{2M}\left(\hat{p}_\varphi - \frac{e\Phi}{2\pi}\right)^2$$

where $\hat{p}_\varphi = -i\hbar\partial_\varphi$ and $\Phi = \pi R^2 B$ is the flux through the ring.



a) What boundary conditions must the wave function $\psi(\varphi)$ obey?

b) Find the exact energy levels $E_n(\Phi)$ and associated eigenfunctions $\psi_n(\varphi)$. Express you answers in terms of the flux through the ring and the *flux quantum* $\Phi_0 = 2\pi\hbar/e$.

c) If the particle is in an eigenstate $\psi_n(\varphi)$ and the flux Φ through the ring is slowly changed, the allowed energy level $E_n(\Phi)$ of the state changes. Does the energy of the particle itself change? And if it does, where does the energy come from?

d) Sketch a graph of the ground-state energy as a function of Φ/Φ_0. For what values of Φ is the ground state degenerate? What are the quantum numbers of the degenerate levels?

e) A uniform electric field $\mathbf{E} = E\mathbf{e}_x$ is now applied. Use perturbation theory to lowest non-vanishing order to find how the energy of the two lowest energy levels varies as a function of Φ in the neighborhood of one of the degeneracy points. Sketch a graph to show how the perturbation alters the graph you drew for part (d).