QM A proton and a neutron are confined in a potential well. Initially we assume that the proton and neutron do not interact with each other, and neglect spin-orbit interactions. Both particles have spin 1/2, so, including the spins, the ground state is four-fold degenerate. To this system we now add an interaction between the spins of the particles given by the Hamiltonian

$$\hat{H} = -k\,\mathbf{S}_{\mathrm{p}} \cdot \mathbf{S}_{\mathrm{n}},$$

where $\mathbf{S}_{\mathrm{p}}$ and $\mathbf{S}_{\mathrm{n}}$ are the spin operators of the proton and neutron, respectively, and k is a positive constant.

a) Of the following operators, $|\mathbf{S}_{\mathrm{p}}|^2$, $|\mathbf{S}_{\mathrm{n}}|^2$, $S_{\mathrm{p},z}$, $S_{\mathrm{n},z}$, $|\mathbf{S}|^2$, S_z, where $\mathbf{S} \equiv \mathbf{S}_{\mathrm{tot}} = \mathbf{S}_{\mathrm{p}} + \mathbf{S}_{\mathrm{n}}$, which commute with $\hat{H}$?

b) Into how many distinct energy levels does the original ground state split in the presence of the interaction $\hat{H}$? Calculate the corresponding energies and state their degeneracy.

We now place the system into a uniform external magnetic field pointing in the positive z-direction, $\mathbf{B} = B\mathbf{e}_z$. The spin-spin interaction $\hat{H}$ continues to be present, and to this is added the effect of the magnetic field described by

$$\hat{H}' = b\,(\mathbf{S}_{\mathrm{p}} + \mathbf{S}_{\mathrm{n}}) \cdot \mathbf{B}$$

where b is a positive constant. (We are here assuming that the g factors for the proton and neutron are equal.)

c) Calculate the corrections to the energies of the states identified in part (b) due to the presence of the magnetic field.

d) Sketch a graph of the energy levels as a function of the external magnetic field strength B_z including the effects of both $\hat{H}$ and $\hat{H}'$. Label the curves with the states you identified in part (b).