QMB Neutrino oscillation occurs because neutrino mass eigenstates are not the same as neutrino flavor eigenstates. When a neutrino is created it is always in a flavor eigenstate — meaning that it is an electron, muon or tau neutrino.

In this problem we will consider only oscillation between the electron neutrino ν_e and the muon neutrino ν_μ.

We will call the mass eigenstates $|\nu_1\rangle$ for mass m_1 and $|\nu_2\rangle$ for mass m_2. The two flavor eigenstates are related to the mass eigenstates *via* a mixing angle θ as

$$\begin{aligned} |\nu_e\rangle &= \cos\theta|\nu_1\rangle + \sin\theta|\nu_2\rangle, \\ |\nu_\mu\rangle &= -\sin\theta|\nu_1\rangle + \cos\theta|\nu_2\rangle. \end{aligned}$$

When an electron neutrino is created in a source, its state $|\psi\rangle$ will be a linear superposition of the two mass eigenstates with masses m_1 and m_2. Depending on how it is created, these mass eigenstates may have different energies E_1, E_2 and corresponding momenta p_1, p_2.

a) What are the amplitudes $\langle x = 0, t = 0, \nu_1|\psi\rangle$ and $\langle x = 0, t = 0, \nu_2|\psi\rangle$ for the electron neutrino to be in each of the m_1 and m_2 mass eigenstates immediately after it is created?

b) The neutrino is detected some time t later and a large distance x from the source. At this distance the space-time part of the wavefunction can be taken to be a unit-amplitude plane wave. What are the amplitudes $\langle x, t, \nu_1|\psi\rangle$ and $\langle x, t, \nu_2|\psi\rangle$?

c) Suppose that $E_1 = E_2$. What is the difference between the momenta p_1 and p_2? (The mass difference is very small so the approximation $\sqrt{1 + x^2} \simeq 1 + x^2/2$ can be used where appropriate).

d) With the $E_1 = E_2$ assumption, what is the phase difference $\Delta\phi$ at the detector between the two amplitudes $\langle x, t, \nu_1|\psi\rangle$ and $\langle x, t, \nu_2|\psi\rangle$? Express your answer in terms of m_1, m_2, E, $\hbar$, the speed of light c and the distance x.

e) Use your answer to part (d) to determine the probability $P_e(x)$ that the detected neutrino remains an electron neutrino, and the probability$P_\mu(x)$ that the neutrino is detected as a muon neutrino?