QMA The wavefunction for the electron in the 1s ground state of the

hydrogen atom is

$$\psi_{1s}(r) = (\pi a_0^3)^{-1/2} \exp\{-r/a_0\},$$

where a_0 is the Bohr radius. This wavefunction was obtained by regarding the proton as a point charge giving a potential $V_0(r) = -e^2/4\pi\epsilon_0 r$. Suppose that, instead of a point charge, the proton charge is distributed uniformly within a sphere of radius r_p.

a) Write down expressions giving the modified interaction potential as seen by the electron both in the region $0 < r < r_p$ and $r > r_p$.

b) Use first order perturbation theory to obtain an integral that gives the shift ΔE_{1s} in the ground state energy due to the finite size of the proton. Without evaluating the integral, state (with reason) as to whether the binding energy of the electron is increased or decreased.

Now we consider how the 1s ground state is affected by the imposition of a constant uniform electric field in the $\hat{z}$ direction. In the absence of the electric field we can write

$$H_0 = E_1|1\rangle\langle1| + E_2|2\rangle\langle2|$$

where $|1\rangle \equiv |\psi_{1s}\rangle$ and $E_1 \equiv E_{1s}$. The perturbation will then be of the form

$$H_1 = \Omega|1\rangle\langle2| + \Omega^*|2\rangle\langle1|$$

where Ω is a (possibly complex) constant that you **do not** have to evaluate.

c) The principal effect of the field is a mixing of the 1s ground state with one or more of the 2s and 2p excited states. By considering the symmetries of these states and that of the perturbation, determine which of the four states $|n = 2, l, m\rangle$ should be taken as $|2\rangle$ to capture the effect.

d) Find the shift in the ground state energy to order $|\Omega|^2$.