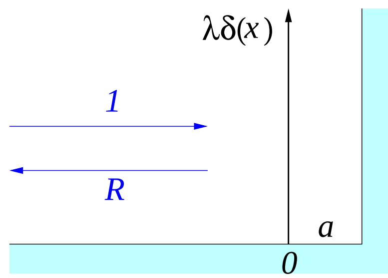
QMA Consider the motion of an electron of mass m in the potential shown in the figure:



$$V(x) = \begin{cases} \lambda\delta(x), & -\infty < x < a, \\ \infty, & x > a. \end{cases}$$

a) Suppose that the delta-function at $x = 0$ is replaced by an impenetrable barrier. Calculate the energy E_0 and wave function of the lowest bound state trapped in the potential well between $x = 0$ and $x = a$.

b) When λ is finite, what boundary conditions should you impose on the wavefunction across the delta-function?

An electron is incident on the potential from the left. In the region $x < 0$ its wavefunction is

$$\psi(x,t) = e^{i(px-Et)/\hbar} + Re^{i(-px-Et)/\hbar}.$$

c) What can we say about the magnitude of R? By matching the wavefunction across the delta function, compute R as a function of E.

d) For finite λ, the bound state that was identified in (a) will no longer be infinitely long lived, but will decay and so have complex energy E_*. What condition should you impose on R to find E_*? Explain why.

e) Assume that $\lambda \gg E_0 a$ and use your condition on R from part (d) to find the approximate energy shift ΔE of the lowest bound state. Also find its lifetime τ (defined by $|\psi|^2 \propto e^{-t/\tau}$). Note that ΔE is of order λ^{-1}, and τ of order λ^2.

Hint: When ϵ is small

$$\cot(\pi + \epsilon) \approx \frac{1}{\epsilon}.$$