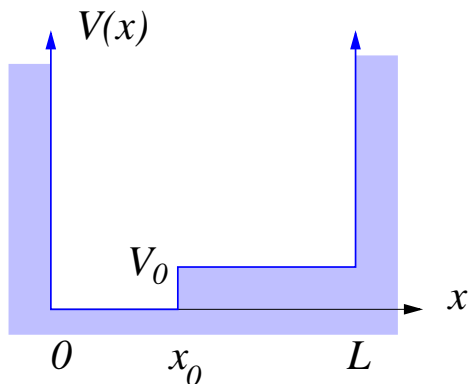


**QM** Consider the quantum mechanics of a particle of mass  $m$  in the one-dimensional potential well with  $V(x) = +\infty$  for  $x < 0$  and  $x > L$ .  $V(x) = 0$  for  $0 \leq x < x_0$ , and  $V(x) = V_0$  for  $x_0 < x \leq L$ .



You will first solve the problem exactly, and then check your answer by comparing with perturbation theory valid when  $V_0 \ll \hbar^2/mL^2$ .

- Draw a qualitative sketch of the wavefunction of the ground state of the system, firstly for  $V_0$  much larger than the ground state energy, and secondly for  $V_0$  much smaller than the ground state energy.
- Write down the Schrödinger equation for this problem, and give all necessary boundary conditions on its solutions. Write down the general form of a solution of energy  $E$  for the case when  $V_0$  is smaller than the energy of the lowest level.
- By applying the boundary conditions to the case in part (b), extract a (transcendental) relation that determines the spectrum of energies in terms of  $V_0, m, \hbar, L, x_0$ . Show that these conditions agree with those of the usual infinite square well when  $V_0 \rightarrow 0$ .
- By expanding to first order in  $V_0$ , find an approximation for all energy levels. You may make use of the expansion

$$\cot(\alpha + \beta) \simeq \cot \alpha - \frac{1}{\sin^2 \alpha} \beta + O(\beta^2)$$

valid for  $\beta \ll \alpha$ .

- Using perturbation theory, compute the first order correction to the energy levels of the infinite square well problem. Does this result agree with your result from part (d)? Does it behave as you would expect when  $x_0 \rightarrow 0$  and  $x_0 \rightarrow L$ ?