QM An electrically charged particle of mass m is in its ground state inside the harmonic oscillator well

$$U(x) = \frac{1}{2} m\omega^2 x^2.$$

The corresponding harmonic-oscillator eigenfunctions are

$$\psi_n(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} e^{-\frac{m\omega}{2\hbar}x^2} H_n\left(x\sqrt{\frac{m\omega}{\hbar}}\right),$$

where $n = 0, 1, 2, \ldots$, $H_0 = 1$, and

$$H_n(\xi) = (-1)^n e^{\xi^2} \frac{d^n}{d\xi^n} e^{-\xi^2}.$$

Suddenly a homogeneous electric field is turned on so the potential energy becomes

$$U_{\text{new}}(x) = \frac{1}{2} m\omega^2 x^2 - Fx.$$

a) What are the new eigenfunctions and energy levels in the new potential?
b) What is the wave function immediately after the field is turned on?
c) Show that the probability that we end up in the new k-th excited state is of the form

$$P_k = \frac{A^k}{k!} e^{-A},$$

where the parameter A is a function of F, m, ω, $\hbar$ that you should find. Does the sum of these probabilities add up to 1? If not, why not?
d) Write the average k-value as a function of F, m, ω and $\hbar$. For small values of F, write approximate expressions for both the average k value and the probability that $k = 0$. Be sure to specify what you mean by small F.

Some useful equations:

$$\begin{aligned}
\int_{-\infty}^{\infty} e^{-x^2+2xx_0} dx &= \sqrt{\pi} e^{x_0^2/4} \\
e^y &= \sum_{k=0}^{\infty} \frac{y^k}{k!} \\
ye^y &= \sum_{k=0}^{\infty} k \frac{y^k}{k!}.
\end{aligned}$$