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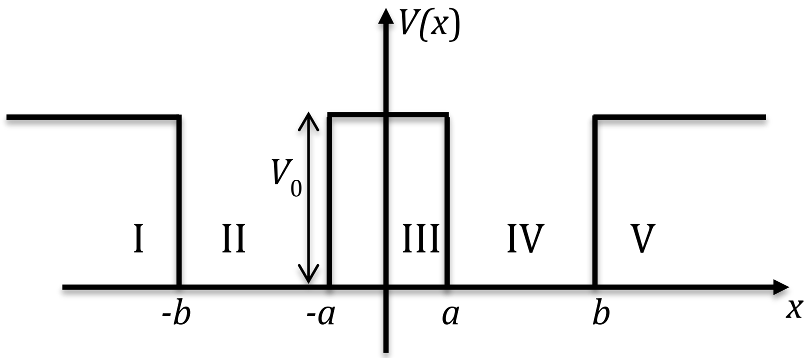
Consider a one-dimensional, symmetric double potential well system (well height V_0; well boundaries at $x = \pm a, \pm b$, as illustrated below), that contains a quantum mechanical particle of energy E. Assume $E < V_0$.



(a) Write down general solutions to the Schrödinger equation in regions I, II, III, IV, and V in terms of amplitudes, E, V_0, x, and fundamental constants. State the boundary conditions necessary for determining the amplitudes. Do **not** solve for the amplitudes.

(b) When the potential wells are well-separated so that we can make a two-state approximation, the basis states Ψ_L and Ψ_R are the groundstate wavefunctions confined to the left and right wells, respectively. In addition, each groundstate has energy E_0.

(i) Sketch the groundstate wavefunctions (first copy the illustration above into your exam books, then make the sketch).

(ii) If the basis states have an interaction energy ΔE, express the Hamiltonian for this "tunneling" system as a 2×2 matrix. Calculate the eigenvalues and eigenvectors for this system.

(c) Sketch the two lowest energy eigenstates of the tunneling system.

(d) Consider the case where the energies of Ψ_L and Ψ_R are changed by a small amount, α and $-\alpha$, respectively. The quantity $\alpha << V_0$, and the basis states still have interaction energy ΔE.

(i) Express the Hamiltonian for this system as a 2×2 matrix.

(ii) Sketch the two lowest energy eigenstates of this system. Is the energy difference between the two lowest energy states bigger, smaller, or the same as the case where $\alpha = 0$?

(e) Going back to the case of unperturbed ($\alpha = 0$) basis states, let's now consider the time-dependence of the system. Let the particle initially be in the left well, so that $\Psi(t=0) = \Psi_L$. Suppose that the particle can tunnel through the potential barrier in a time τ. Estimate τ in terms of ΔE.

(f) Now explicitly calculate the time-dependence of $\Psi_L(t)$ to show that the particle eventually returns to its initial state (at least, to within a phase factor).