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An electron of charge $-|e|$ and mass m moves in the presence of a uniform magnetic field pointing in the *z*-direction $\boldsymbol{B} = B\hat{\boldsymbol{z}}$. The motion of the electron is confined to the xy - plane.

(a) As a warm up, write down the Hamiltonian, the energy eigenvalues, and the position-momentum commutation relation for a one dimensional harmonic oscillator.

(b) Show that the commutation relation for the kinetic momentum $\boldsymbol{\Pi} = \mathbf{p} - \frac{e}{c}\boldsymbol{A}$, is given by

$$[\Pi_x, \Pi_y] = \frac{i\,\hbar\,e}{c}\,B$$

where $\mathbf{p}$ is the canonical momentum, and $\boldsymbol{A}$ is the magnetic vector potential.

(c) The Hamiltonian for this problem is

$$H = \frac{\Pi_x^2 + \Pi_y^2}{2m}$$

By comparing your results from parts (a) and (b) or otherwise, compute the energy eigenvalues. Express the characteristic angular frequency of the electron motion in terms of the parameters of the problem.

(d) Using the gauge $A_z = A_y = 0$, and $A_x = -By$, show that the eigenfunctions can be expressed as

$$\psi_n(x, y) = e^{-ik_x x}\, f_n(y - y_0)$$

Here, n is the quantum number of the n^{th} energy eigenstate, and $f_n(y)$ is a function that satisfies the Schrödinger equation. You do not need to specify the function $f_n(y)$; however you do need to find y_0.

(e) Assume that the electrons are confined to a region with dimensions L_x and L_y with periodic boundary conditions in the x direction. In terms of B, L_x, L_y, and the quantity $\phi_0 = hc/e$, compute the degeneracy of each energy level. Assume that B has been adjusted to guarantee that the degeneracy is an integer.