# 4

A proton and a neutron are confined by a three-dimensional potential. For this problem assume that the proton and neutron do not interact with each other, and neglect spin-orbit interactions. Both particles have spin 1/2. Including the spins, the ground state is four-fold degenerate.

To this sytem we now add the interaction between the magnetic dipole moments of the particles described by the interaction Hamiltonian:

$$H' = -k\, \boldsymbol{S}_p \cdot \boldsymbol{S}_n \ , \tag{1}$$

where $\boldsymbol{S}_p$, $\boldsymbol{S}_n$ are the spin operators of the proton and neutron, respectively, and $k$ is a positive constant.

(a) Consider the following operators:

$$S_p^2\, , S_n^2\, , S_{pz}\, , S_{nz}\, , S^2\, , S_z\ ,$$

where $\boldsymbol{S} = \boldsymbol{S}_p + \boldsymbol{S}_n$. State which of these operators commute with $H'$.

(b) Into how many distinct energy levels does the original ground state split in the presence of $H'$? Calculate the corresponding energies and state their degeneracy.

We now place the system into a uniform external magnetic field, which points in the positive $z$-direction, $\mathbf{B} = B_z\hat{z}$. The spin-spin interaction described by $H'$ in Eqn. (1) continues to be present and the additional interaction Hamiltonian is:

$$H'_B = b\,(S_{pz} + S_{nz})\, B_z\ , \tag{2}$$

where $b$ is a positive constant. (Note: We used the approximation that the $g$ factors for the proton and the neutron are equal.)

(c) Calculate the corrections to the energies of the states identified in part (b) due to the presence of the magnetic field.

(d) Sketch a graph of the energy levels as a function of the external magnetic field strength, $B_z$, including the effects of both $H'$ and $H'_B$. Identify the curves with the corresponding states identified in part (b).