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A isolated light-responsive molecule has an excited state whose energy above the ground state is E_0. Consider two of these molecules that are no longer isolated, but lie close to each other, being separated by a small distance R. We will label one of them D for *donor*, and the other A for *acceptor*. We consider only the states $|1\rangle \equiv |D^*, A\rangle$ and $|2\rangle \equiv |D, A^*\rangle$ of the two-molecule system. Here the $*$ denotes that the labelled molecule is in its excited state. The hamiltonian for the combined two-molecule system has matrix elements

$$\langle 1|H|1\rangle = \langle 2|H|2\rangle = E_0, \quad \langle 1|H|2\rangle = \langle 2|H|1\rangle = V.$$

All other states of the system, and any matrix elements between them, can be ignored.

a) Find the energy eigenvalues of the two-molecule system. Express you answer in terms of E_0 and V.

b) Find the energy eigenstates of the two-molecule system as a linear combination of $|1\rangle$ and $|2\rangle$.

c) At time $t = 0$, only the donor molecule is excited. The two-molecule system is therefore in state $|1\rangle$. What is the earliest time t at which the excitation energy is entirely transferred to the acceptor molecule A?

d) The off-diagonal matrix element

$$V = \langle 2|H|1\rangle = \langle D, A^*|H|D^*, A\rangle$$

arises from the dipole-dipole interaction between the molecules, and is given (in SI units) by

$$V = \langle D, A^*|H|D^*, A\rangle = \frac{\boldsymbol{\mu}_D \cdot \boldsymbol{\mu}_A}{4\pi\epsilon_0 R^3} = \frac{e^2}{4\pi\epsilon_0 R^3}\langle D|\mathbf{r}_D|D^*\rangle \cdot \langle A^*|\mathbf{r}_A|A\rangle.$$

In $\boldsymbol{\mu}_D = e\langle D|\mathbf{r}_D|D^*\rangle$ the vector $\mathbf{r}_D$ is the displacement of an electron from the center of charge of molecule D. Similarly $\mathbf{r}_A$ is the displacement of an electron in molecule A. The "$\cdot$" in the rightmost expression indicates a scalar product between the two displacement vectors. It is reasonable to conjecture that $|\boldsymbol{\mu}|/e$ is smaller than the size of the molecule. Given that the energy transfer between the donor D and acceptor A takes about 1 ns when $R = 5$ nm, use your answer to part (c) to estimate $|\boldsymbol{\mu}|/e$ and compare it with the size (roughly 1 nm) of a light-responsive molecule. Is the conjecture about the magnitude of $|\boldsymbol{\mu}|/e$ supported by your estimate?