Imagine you did not know thc Hydrogcn-atom cncrgics and wavefunctions.

(a) Use the variational technique to estimate the ground-state energy of Hydrogen. Use as your trial wavefunction a Gaussian in the radial coordinate r (the distance between the electron and the proton)

$$\psi(r) = Ae^{-br^2}$$

where b and A are real, positive constants. Let the constant b be the variational parameter.

(b) What are the features of this trial wavefunction that make it an appropriate choice?

(c) Write down an appropriate trial wavefunction $\psi(r, \theta, \varphi)$ (where r, θ, φ are the usual spherical coordinates) that could be used to estimate the energy of the lowest orbital angular momentum $l = 1$ state of Hydrogen via the variational technique (do not actually do the calculation). What are the features of your trial wavefunction that make it an appropriate choice?

Some useful integrals:

$$\int_0^\infty dx\, e^{-cx^2} = \frac{1}{2}\sqrt{\frac{\pi}{c}}$$

$$\int_0^\infty dx\, xe^{-cx^2} = \frac{1}{2c}$$

$$\int_0^\infty dx\, x^n e^{-cx^2} = \frac{1}{2}\left(\frac{1}{2c}\right)^{n/2}(n-1)(n-3)(n-5)...1\sqrt{\frac{\pi}{c}} \quad (n \text{ even})$$

$$= \frac{1}{2}\left(\frac{1}{c}\right)^{(n+1)/2}((n-1)/2)! \quad (n \text{ odd})$$