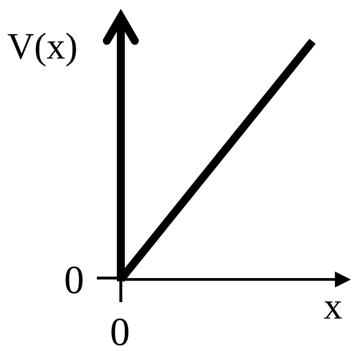
This problem uses the variational principle to calculate an upper bound for the ground state energy of a one dimensional quantum particle confined to a potential given by

$$V(x) = \begin{cases} \infty & for\ \ x < 0 \\ \lambda x & for\ \ x \geq 0 \end{cases}$$

This potential is sketched in the figure below.



Use a function proportional to $xe^{-\alpha x}$ as the variational wave function. The integral $\int_0^\infty dx\, x^n e^{-\beta x} = \frac{n!}{\beta^{n+1}}$ may be of use in solving this problem.

(a) What is the normalization factor for this wave function?

(b) Find an upper bound for the ground state energy.

(c) Copy the shape of the potential function in your answer book and draw a line to indicate the ground state energy. On the same graph sketch the shape of the wave function.

(d) Now extend this problem to 3 dimensions in the following manner: In the y and z directions the particle is free to move anywhere between $\pm\infty$, without forces. What are the quantum numbers of the energy eigenstates? Which are continuous and which are discrete? Ignore spin.