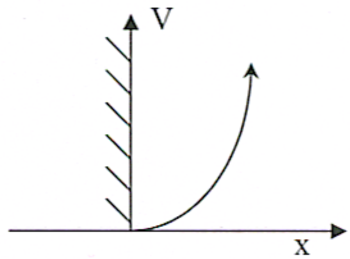
The (non-normalized) eigenfunctions for the familiar one-dimensional simple harmonic oscillator (SHO) can be expressed as follows:

$$\psi_0 \propto e^{-\frac{x^2}{a^2}} \qquad \psi_1 \propto x\, e^{-\frac{x^2}{a^2}} \qquad \psi_2 \propto \left(1-4\frac{x^2}{a^2}\right) e^{-\frac{x^2}{a^2}}$$

$$\psi_3 \propto \left(3\frac{x}{a}-4\frac{x^3}{a^3}\right) e^{-\frac{x^2}{a^2}} \qquad \psi_4 \propto \left(3-24\frac{x^2}{a^2}+16\frac{x^4}{a^4}\right) e^{-\frac{x^2}{a^2}}$$



Now consider an electron in a 1-dimensional half-harmonic potential, as shown here, defined by:

$V(x) = K\,x^2/2$ for $x > 0$;

$V(x) = \infty$ for $x \le 0$.

(a) Sketch the ground and first excited state for this new potential. (Hint: It may help you to draw the usual SHO states first.)

(b) Write a normalized wavefunction for the ground state, in terms of m (the electron mass) and ω (the frequency of a classical SHO with the same 'spring constant' K); be sure to solve for the constant 'a' in the expressions above.

(c) What are the eigen-energies for this new potential (write your answer in terms of 'a')?

(d) Now we add a constant electric field $\vec{E} = E_0\hat{x}$. Use first-order perturbation theory to estimate the new ground state energy (again in terms of 'a').

(e) Set $\vec{E} = 0$. Now add in a second electron. Ignoring the electrical interaction between the electrons, write down both the total energy and the spatial part of the new lowest-energy 2-particle wavefunction (you can ignore normalization now), assuming the electrons are in a *singlet* spin state.

(f) Repeat part (e), assuming the electrons are instead in a *triplet* spin state.

(g) If we now "turn on" the electron-electron interaction, for each of the above states (e) and (f), state whether the energy is increased or decreased. Which of the states has the larger shift? Briefly explain why.