[redacted]

Consider a system consisting of a spin-1/2 particle with a gyromagnetic ratio, γ, placed in a uniform magnetic field. The magnetic field is given as

$$\vec{B} = (B_x, 0, B_z),$$

where B_x and B_z are the x and z components, respectively. B_x and B_z are constants and do not vary with time. Note that the gyromagnetic ratio, γ, is defined as

$$\vec{\mu} = \gamma \vec{S},$$

where $\vec{\mu}$ and $\vec{S}$ are the magnetic moment and the spin operator, respectively. Note that $\vec{S}$ for a spin-1/2 particle can be expressed in terms of the Pauli operators:

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

The Hamiltonian that describes the interaction of this spin-1/2 particle with the magnetic field is

$$H = -\vec{\mu} \cdot \vec{B}.$$

(a) What are the energy eigenvalues of this system?

(b) Find the energy eigenstates of this system.

(c) At $t = 0$, the spin-1/2 particle is prepared with its spin pointing along the positive z direction. What is the probability of finding the particle's spin pointing along the negative z direction for $t > 0$?

(d) Show by giving a derivation that the time evolution of the spin is described by the equation

$$\frac{d\vec{S}}{dt} = \gamma \vec{S} \times \vec{B}.$$

[This equation is particularly easy to derive in the Heisenberg representation, but you may derive it in any representation you wish.]

(e) Draw a diagram that shows qualitatively the motion of $\vec{S}$ relative to $\vec{B}$, when $\vec{S}$ is not colinear with $\vec{B}$, in the classical limit.