A cesium atom used in an atomic clock is driven by a classical, oscillating magnetic field between two hyperfine states $|a\rangle$ and $|b\rangle$. The Hamiltonian which describes the atom interacting with the classical field is:

$$H = \frac{\hbar w_0}{2}\left(|a\rangle\langle a| - |b\rangle\langle b|\right) + \hbar 2\Omega\cos\left(w_0 t\right)\left(|a\rangle\langle b| + |b\rangle\langle a|\right)$$

where the "coupling" Ω is a real number.

(a) One can write the atomic wavefunction as

$$y\,(t) = c_a(t)|a\rangle + c_b(t)|b\rangle .$$

Using Schrödinger's equation, write down differential equations for the coefficients $c_a(t)$ and $c_b(t)$.

(b) At the beginning of an experiment, the atom is prepared in state $|a\rangle$ at time t=0. Defining the coefficients $c_a' = e^{i\frac{w_0}{2}t} c_a$ and $c_b' = e^{-i\frac{w_0}{2}t} c_b$, show that the coefficients c_a' and c_b' satisfy the simultaneous differential equations

$$i\dot{c}_a' = \Omega c_b'$$

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Work in the $\Omega \ll w_0$ limit and ignore oscillations in the amplitudes $c_a(t)$ and $c_b(t)$ at frequencies much higher than Ω. Hence calculate $c_a(t)$ and $c_b(t)$.

(c) After how much time is the atom 100% likely to be measured in state $|b\rangle$?

(d) If the phase of the classical field is changed so that the Hamiltonian is

$$H = \frac{\hbar w_0}{2}\left(|a\rangle\langle a| - |b\rangle\langle b|\right) + \hbar 2\Omega\cos\left(w_0 t + f\right)\left(|a\rangle\langle b| + |b\rangle\langle a|\right)$$

and the experiment is repeated, does your answer to part (c) change? Why or why not?