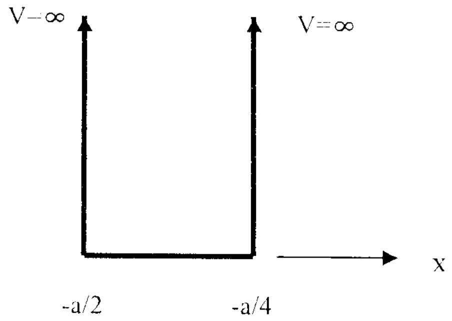
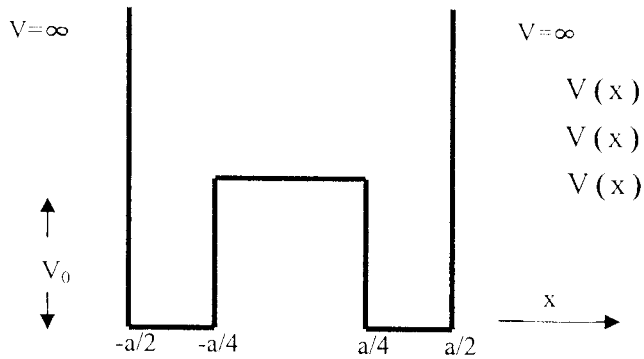
Consider a particle of mass m in an infinitely deep single one dimensional square well potential.



a) Determine the unnormalized eigenfunction $\Psi_0(x)$ and eigenenergy ε_0 of the ground state.

Now consider the particle in an infinitely deep one dimensional square well potential with a potential barrier V_0 as shown. $V(x) = \infty$ for $a/2 < |x|$; $V(x) = 0$ for $a/4 < |x| < a/2$; $V(x) = V_0$ for $|x| < a/4$; Assume $V_0 >> h^2/ma^2$

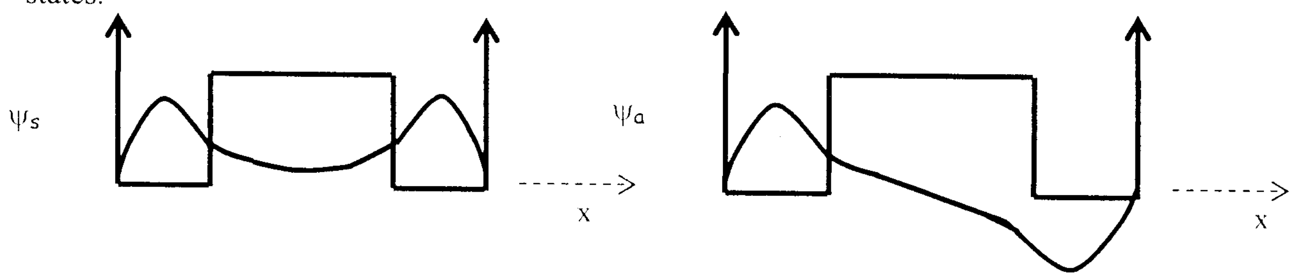


$V(x) = \infty \qquad \frac{a}{2} < |x|$

$V(x) = 0 \qquad \frac{a}{4} < |x| < \frac{a}{2}$

$V(x) = V_0 \qquad |x| < \frac{a}{4}$

Let ψ_{0L} and ψ_{0R} be the eigenfunctions of the ground state in the left and in the right wells respectively, neglecting quantum mechanical tunneling or any other coupling between the wells. Assume the two lowest eigenfunctions for the double well system (with coupling) ψ_s and ψ_a are as shown below. These are referred to as the bonding (symmetric) and anti-bonding (antisymmetric) states.



b) Which one of the wave functions (ψ_s or ψ_a) is the ground state of the double well system? Explain briefly your answer.

c) Write down an approximation for the unnormalized eigenfunction ψ_s as a linear combination of ψ_{0L} and ψ_{0R} and determine the corresponding approximate eigenenergy E_s.

d) Write down an approximation for the unnormalized eigenfunction ψ_a as a linear combination of ψ_{0L} and ψ_{0R} and determine the corresponding approximate eigenenergy E_a

The system is prepared at time $t = 0$ such that the particle is in the ψ_{0L} state (i.e., in the left well only). The system is then left to evolve in time. Due to quantum mechanical tunneling, which was previously neglected, the particle state evolves to ψ_{0R} for the first time at $t = T$.

e) Determine, in terms of T, the difference between the <u>exact</u> eigenvalues of the bonding and antibonding states $E_s - E_a$.